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An overview on *disk heating*

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§1 Galaxies as collisionless systems

We can analyze the movements of the stars of a galactic population and see that they are collisionless: both geometrical and strong encounters are negligible, with a relaxation time greater than Hubble time, while weak encounters could be important in globular cluster and open cluster systems.

Let's analyze the encounters of the stars, considered as point masses, starting from the configuration in Figure 1a, with a star with mass M moving with $\vec{v}$ velocity along the x axis, assuming that another star with mass m is steady; the distance $b = r \cos \theta$ between the x axis and m is called *impact parameter*, and the component $\vec{F}_\perp$ of the gravitational force perpendicular to x axis is

$$F_\perp = M \frac{dv_\perp}{dt} \quad [1]$$

$$F_\perp = F \cos \theta = \frac{GMm}{r^2} \cos \theta = \frac{GMm}{r^2} \frac{b}{r} = \frac{GMm}{(x^2 + b^2)^{3/2}} = \frac{GMm}{b^2} \frac{1}{\left[1 + \left(\frac{x}{b}\right)^2\right]^{3/2}} \approx \frac{GMm}{b^2} \frac{1}{\left[1 + \left(\frac{vt}{b}\right)^2\right]^{3/2}}$$

therefore, from [1] and the latter relation we have $\frac{dv_\perp}{dt} = \frac{Gm}{b^2} \frac{1}{\left[1 + (vt/b)^2\right]^{3/2}}$, and integrating

with the change of variable $z = vt/b$ ($t = bz/v$, $dt = b/v dz$), we get

$$v_\perp = \frac{Gm}{bv} \int_{-\infty}^{+\infty} \frac{dz}{[1 + z^2]^{3/2}} = \frac{2 Gm}{bv} \left[\frac{z}{(1 + z^2)^{1/2}} \right]_0^{+\infty} = 2 \frac{Gm}{bv} \left[\left[\frac{1}{(1 + 1/z^2)^{1/2}} \right]_{+\infty} - \left[\frac{z}{(1 + z^2)^{1/2}} \right]_0 \right].$$

Thus
$$v_{\perp} = 2 \frac{Gm}{bv} . \quad [2a]$$

We have large deflections and then *strong encounters* when $v < v_{\perp}$ or

$$b_s = 2 \frac{Gm}{v v_{\perp}} \geq b , \quad [2b]$$

where the threshold b_s is the impact parameter for strong encounters.

We have *weak encounters* when $v \geq v_{\perp}$.

Therefore $|v_{\perp}|$ is roughly equal to the force of closest approach, Gm/b^2 , times the duration of this force b/v .

Figure 1a

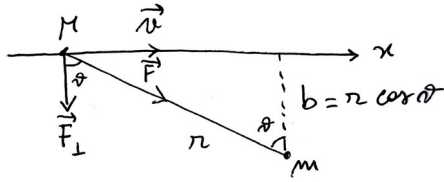
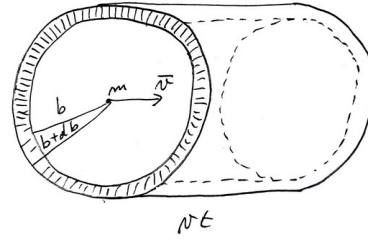


Figure 1b



Looking for the cumulative effects of all the distant stars on our test star m , we consider the total squared perpendicular velocity, because the average value $\langle v_{\perp} \rangle$ is zero, due to the sum of all the contributions of the random movements of the stars of the system.

Thus
$$(v_{\perp}^2)_{\text{tot}} \approx N v_{\perp}^2 . \quad [3]$$

To evaluate a volume at radius $b + db$, like a pipeline, greater than the impact parameter b , and then to analyze the weak encounters as shown in (Figure 1b), we get $dV = 2 \pi b db v t$ and therefore, from [3] and [2a] for $v_{\perp}^2$ we have

$$(v_{\perp}^2)_{\text{tot}} \approx \int_{b_{\min}}^{b_{\max}} dN v_{\perp}^2 = \int_{b_{\min}}^{b_{\max}} n 2 \pi b db v t \left(\frac{2 G m}{b v} \right)^2 = \frac{8 \pi G^2 m^2 n t}{v} \int_{b_{\min}}^{b_{\max}} \frac{db}{b} = 8 \pi G^2 m^2 n t \ln(\Lambda)$$

where n is the number density of stars, and $\ln(\Lambda) = \ln(b_{\max}/b_{\min})$ is the logarithm of the ratio between the maximum and minimum impact parameter, called the *Coulomb logarithm*, a measure of the distance at closest approach.

We can consider $b_{\max} = R$ as the size of the galaxy, then we get R from the virial theorem

$$v^2 = \frac{GNm}{R} , \quad [4]$$

and $b_{\min} = b_s = 2 GM/v^2$ from the threshold for strong encounters in [2b], therefore we have

$$\ln(\Lambda) = \ln\left(\frac{R}{b_{\min}}\right) = \ln\left(\frac{GNm}{v^2} \frac{v^2}{2Gm}\right) = \ln\left(\frac{N}{2}\right) \approx \ln(N) .$$

Thus we obtain the weak encounters threshold when

$$v_{\perp}^2 = \frac{8 \pi G^2 m^2 n t}{v} \ln(N) .$$

As soon as we have this threshold we have the time scale for the weak encounters

$$t_w = \frac{v^3}{8 \pi G^2 m^2 n \ln(N)} . \quad [5]$$

We can compare the scale time for weak encounters to the crossing scale time of the system $t_c = 2R/v$, that is the number of times the star crosses the system to have a significative deviation from the orbit. From [4] and [5] we get

$$\frac{t_w}{t_c} = \frac{v^3}{8 \pi G^2 m^2 n \ln(N)} \frac{1}{\frac{2R}{v}} = \left(\frac{GNm}{R} \right)^2 \frac{1}{16 \pi G^2 m^2 \frac{N}{\frac{4}{3} \pi R^3} R \ln(N)} = \frac{N}{12 \ln(N)}$$

which depends only on the number of the stars.

The more stars we consider, the longer it takes to have a weak encounter, a time that we call *relaxation time*. With this relation it's easy to see that with the global clusters, and especially with the open clusters, this ratio becomes critical, so: while galaxies are *collisionless systems*, in clusters systems the encounters could play a key role.

By studying the general case of galactic encounters, it's possible to apply the *Chandrasekhar dynamical friction formula* (originally derived for a uniform background distribution) to a body of mass M_{sat} orbiting in a spherical system with density $\rho(r)$ and total mass $M_{\text{host}} \gg M_{\text{sat}}$. Let us assume that the mass density distribution of the host system is that of a *singular isothermal sphere*, suitable for studying the dynamics of stellar flows, and obtain this mass density equation starting from a *distribution function* $f(\epsilon)$ (see below) like the following

$$f(\epsilon) = \frac{\rho_1}{(2 \pi \sigma^2)^{3/2}} e^{-\frac{\epsilon}{\sigma^2}} ,$$

where σ is a parameter which we will identify as dispersion velocity, ρ_1 is a scale density parameter, $\epsilon \equiv \psi - \frac{1}{2} v^2$ is defined as relative energy with ψ as potential (in the energy formulas we consider unity masses).

The mass density is proportional to the volumetric number density of stars $\rho = \int f d^3 \vec{v}$, where $f(\vec{x}, \vec{v}, t)$ is the distribution function (or phase space density) defined in such way that $f(\vec{x}, \vec{v}, t) d^3 \vec{x} d^3 \vec{v}$ gives us the number of stars within $d^3 \vec{x}$ with velocity within $d^3 \vec{v}$.

Therefore we obtain

$$\rho = \int_0^\infty f d^3 \vec{v} = 4 \pi \int_0^\infty f(\epsilon) v^2 dv = \frac{4 \pi \rho_1}{(2 \pi \sigma^2)^{3/2}} e^{\frac{\psi}{\sigma^2}} \int_0^\infty e^{-\frac{v^2}{2 \sigma^2}} v^2 dv .$$

Substituting $t^2 = v^2/2 \sigma^2$ and $dv = \sqrt{2} \sigma dt$ in the last term, we get

$$\rho = \frac{4 \rho_1}{\sqrt{\pi}} e^{\frac{\psi}{\sigma^2}} \int_0^\infty e^{-t^2} t^2 dt = \frac{4 \rho_1}{\sqrt{\pi}} e^{\frac{\psi}{\sigma^2}} \frac{1}{2} \frac{\sqrt{\pi}}{2} \quad \text{that is} \quad \rho = \rho_1 e^{\frac{\psi}{\sigma^2}} .$$

If we rewrite it as $\psi = \sigma^2 \ln\left(\frac{\rho}{\rho_1}\right) = \sigma^2 \ln(\rho) - \sigma^2 \ln(\rho_1)$, differentiating we get $d\psi = \sigma^2 d \ln(\rho)$.

Then we can rewrite the Poisson equation $\nabla^2 \Phi = 4 \pi G \rho$ for the potential as

$$\begin{aligned} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\psi}{dr} \right) &= -4\pi G \rho \quad \text{that is} \\ \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d \ln(\rho)}{dr} \right) &= -\frac{4\pi G}{\sigma^2} \rho . \end{aligned} \quad [6]$$

We are looking for a solution in the form $\rho = c r^{-b}$, therefore we rewrite it as $\ln(\rho) = \ln(c) - b \ln(r)$, and differentiate it as $d \ln(\rho) = -b d \ln(r)$, then we apply these values to the Poisson equation [6], obtaining the two parameters of the exponential as $b = 2$, and $c = \frac{\sigma^2 b}{2\pi G}$, and finally the mass density equation:

$$\rho(r) = \frac{\sigma^2}{2\pi G} \frac{1}{r^2} = \frac{v_c^2}{4\pi G} \frac{1}{r^2}, \quad [7]$$

with the parameter $\sigma = v_c/\sqrt{2}$, that now is clear to be a velocity dispersion, obtained from

$$v_c^2(r) = \frac{G}{r} M(r) = \frac{G}{r} \int_0^r 4\pi \hat{r}^2 \rho d\hat{r}^2 = \frac{G}{r} \int_0^r 4\pi \hat{r}^2 \frac{\sigma^2}{2\pi G \hat{r}^2} d\hat{r} = \frac{G}{r} \frac{2\sigma^2}{G} r ,$$

substituting the value ρ from the first term of [7], where v_c is the (radius independent) circular velocity.

In this case, for a satellite of mass M_{sat} orbiting on a circular orbit (with $v = v_c$) at radius r , following (Cimatti et al., 2020), the Chandrasekhar dynamical friction formula reads

$$\frac{dv}{dt} = -\text{const} \ln(\Lambda) \frac{G M_{\text{sat}}}{r^2} = -2\pi \text{const} \ln(\Lambda) \frac{G^2 \rho M_{\text{sat}}}{\sigma^2},$$

where the *constant* comes from the statistical *error function*.

Therefore the dynamical friction timescale

$$t_{\text{fric}} \equiv \frac{v}{|dv/dt|} = \frac{1}{\sqrt{2}\pi \text{const} \ln(\Lambda)} \frac{\sigma^3}{G^2 \rho M_{\text{sat}}} \quad [8]$$

is inversely proportional to M_{sat} and ρ , for given velocity dispersion of the host.

Applying [7] to [8] we obtain:

$$t_{\text{fric}} = \frac{\sqrt{2}}{\text{const} \ln(\Lambda)} \frac{\sigma r^2}{G M_{\text{sat}}} .$$

This means that in a time span of a few Gyr, for instance with the values of our assumptions applied to the Milky Way, the orbit of a massive satellite has substantially reduced, while the orbit of lower mass satellites (such as globular clusters) has not been significantly modified by dynamic friction.

§2 Jeans equations and the velocity ellipsoid

In our galaxy sample, in the absence of encounters stars, we can consider the stars movement like that of a fluid, with a flow described by a continuity equation in the phase space $(\vec{x}, \vec{v}, t)$, the **first Jeans equation**, analogous to that satisfied by the density $\rho(x, t)$ of an ordinary fluid flow (with v instead of ρ), which reads:

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\partial}{\partial x_i} (\mathbf{v} \overline{v_i}) = 0 .$$

We have also a **second Jeans equation**, which, in the stellar dynamics, corresponds to the motion equation in fluid dynamics (Eulero equation); like the first Jeans equation, it derives from the *collisionless Boltzman equation* (CBE):

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i} v_i - \frac{\partial f}{\partial v_i} \frac{\partial \Phi}{\partial x_i} = 0 ;$$

it is worth noting that this equation means that the flow of stellar phase point through phase space is incompressible and the phase space density f around the phase point of a given star always remains the same.

The formula of the second Jeans equation it's focused on the galactic characteristic of the fluid, and all the quantities are observables; in cartesian coordinates, it reads:

$$\mathbf{v} \frac{\partial \overline{v_j}}{\partial t} + \mathbf{v} \overline{v_i} \frac{\partial \overline{v_j}}{\partial x_i} = - \mathbf{v} \frac{\partial \Phi}{\partial x_j} - \frac{\partial}{\partial x_i} (\mathbf{v} \sigma_{ij}^2) ,$$

where $\overline{v_i} = \frac{1}{V} \int f v_i d^3 \vec{v}$ is the stars mean velocity, and $\sigma_{ij}^2 = \frac{1}{V} \int f (v_i - \overline{v_i})(v_j - \overline{v_j}) d^3 \vec{v}$ is the *variance* which indicates the *velocity dispersion* (that is the statistical velocity dispersion of velocities about the mean velocity) which we can consider as $\sigma_{ij}^2 = \overline{v_i v_j} - \overline{v_i} \overline{v_j}$, where $\mathbf{v} \sigma_{ij}^2$ is called **stress tensor**, that describes an anisotropic pressure.

We may choose a set of orthogonal axes in which σ^2 is diagonal, that is $\sigma_{ij}^2 = \sigma_{ii}^2 \delta_{ij}$, then define as **velocity ellipsoid** the one with semiaxes σ_{11} , σ_{22} and σ_{33} , triaxial or simmetric.

In a cylindrical system of coordinates (R, φ, z) the velocity dispersions $\sigma_R^2, \sigma_\varphi^2, \sigma_z^2$ define the main axes of the velocities ellipsoid. The ellipsoid is parametrized by its two axes ratios σ_φ/σ_R and σ_z/σ_R , and in an axisymmetrical disk (that is, a system in which the symmetry is represented by the axis of rotation, usually the z axis), with stellar orbits not too far from circular (the epicycle approximation, see §4), σ_φ/σ_R depends only on the circular velocity and not on any disk heating mechanism, while the ratio σ_z/σ_R can be used to probe the velocity distribution and therefore to constrain the heating processes in the galactic disk.

Few authors (Westfall et al., 2005-2008, see further on §3) have found some evidences for variation in the velocity ellipsoid ratio σ_z/σ_R with radius, a circumstance to be studied further.

For completeness, the II Jeans equation in cylindrical coordinates reads:

$$\frac{\partial}{\partial t} (\mathbf{v} \overline{v_R}) + \frac{\partial}{\partial R} (\mathbf{v} \overline{v_R^2}) + \frac{\partial}{\partial z} (\mathbf{v} \overline{v_R v_z}) + \mathbf{v} \left(\frac{\overline{v_R^2} - \overline{v_\varphi^2}}{R} + \frac{\partial \Phi}{\partial R} \right) = 0 ,$$

the II Jeans equation in spherical coordinates reads:

$$\frac{d}{dr} (\mathbf{v} \overline{v_r^2}) + \frac{\mathbf{v}}{r} \left[2 \overline{v_r^2} - (\overline{v_\varphi^2} + \overline{v_\theta^2}) \right] = - \mathbf{v} \frac{d\Phi}{dr} .$$

Another useful equation (using observable parameters) is the **asymmetric drift** equation (defined as the circular velocity minus the azimuthal velocity $v_a = v_c - \overline{v_\phi}$, which formalizes the tendency of the orbiting stars to have an orbital velocity smaller than the Standard of Rest velocity, the circular velocity), which in a cylindrical coordinates system (R, ϕ, z) , valid for nearly circular orbits, reads:

$$v_c^2 - \overline{v_\phi^2} = -\sigma_R^2 \left[1 - \frac{\sigma_\phi^2}{\sigma_R^2} + \frac{R}{v \sigma_R^2} \frac{\partial}{\partial R} (v \sigma_R^2) \right] = -\sigma_R^2 \left[\beta + \frac{\partial \ln(v \sigma_R^2)}{\partial \ln(R)} \right], \quad [9]$$

where the *anisotropic parameter* $\beta \equiv 1 - \frac{\sigma_\phi^2}{\sigma_R^2}$ is null if we assume the isotropy.

§3 Analysis of the projected kinematic and the velocity ellipsoid

In a galaxy other than the Milky Way, the first measurements of the so called **disk heating**, that is the increase of the velocity dispersion in the inner stars of a (rotating) galaxy (along the major and the minor axes of the same galaxy as seen in the sky plane), were made by Gerssen et al. in a paper of 1997, studying the *SA(r)b* galaxy *NGC488* (de Vaucouleurs morphology classification from NED database: spiral galaxy not barred with a ring and a medium opening and resolution of the spiral arms).

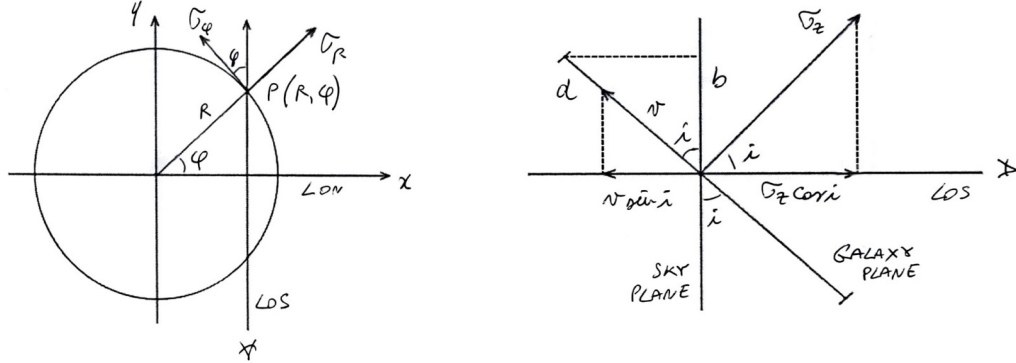
They studied the molecular clouds trying to explain the disk heating processes they observed in the galaxy, considering the CO molecule as optical spectroscopic tracer of molecular hydrogen of the star forming clouds.

The atomic hydrogen can be revealed with radio telescopes from the emission at *21 cm*, due to a fine transition of the electrons that change their spin (parallel - antiparallel) with respect to the nucleus spin. Both atomic and molecular hydrogen are the fundamental components of the proto-clouds from which the stars are born. From the most abundant atomic hydrogen can form the colder molecular hydrogen, whose clouds (also composed of some other molecules, such as carbon monoxide and others) collapse and, if they exceed the mass of Jeans limit, will become proto stars. This Jeans mass limit is proportional to $T^{3/2}$ and $\rho^{-1/2}$, so the stars will form more easily from colder and more dense clouds.

In spiral galaxies, the star formation regions are located on the galactic plane, where we can find molecular clouds in the thickness of a few tens of parsecs, as we can also see in our Milky Way in IR images (which can see beyond the dust clouds), while the stellar region is hundreds of parsecs thick.

To further study the disk heating, we will follow the steps of (Gerssen et al., 1997), in the analysis of galaxy data and disk heating and the paper of (Shapiro et al., 2003) that associate the disk heating as a function of Hubble type galaxies, then we will analyze some hypothesis on the causes of disk heating, after the §4 dedicated to epicycle approximation in axisymmetrical systems.

Figure 2



In Figure 2 (where the x axis is on the line of nodes, the intersection between the sky plane and the galaxy plane) we consider a rotating spiral galaxy with an i angle inclination to the sky plane (galaxy is face on with no tilt if $i = 0$), therefore an observer see the galaxy as an ellipse, with the major axis a equal to the galaxy radius R and the minor axis b in the relation $b = R \cos(i)$ with it, so that we can get the tilt angle as $i = \arccos(b/R)$.

It's possible to define the velocities ellipsoid in any point $P(R, \varphi)$ of the disk plane, starting from the measure of the velocities dispersion projected along the line of sight (with reference to Figure 2):

$$\begin{aligned} \text{radial} \quad \sigma_{R \text{ LOS}} &= \sigma_R \sin(\varphi) \\ \text{tangential} \quad \sigma_{\varphi \text{ LOS}} &= \sigma_\varphi \cos(\varphi) \end{aligned}$$

and then, considering the tilt of the galaxy plane:

$$\begin{aligned} \sigma_{R \text{ LOS}} &= \sigma_R \sin(\varphi) \sin(i) \\ \sigma_{\varphi \text{ LOS}} &= \sigma_\varphi \cos(\varphi) \sin(i) \\ \sigma_{z \text{ LOS}} &= \cos(i) \end{aligned}$$

therefore
$$\sigma_{\text{LOS}}^2 = \sigma_R^2 \sin^2(\varphi) \sin^2(i) + \sigma_\varphi^2 \cos^2(\varphi) \sin^2(i) + \sigma_z^2 \cos^2(i)$$

that is
$$\sigma_{\text{LOS}}^2 = [\sigma_R^2 \sin^2(\varphi) + \sigma_\varphi^2 \cos^2(\varphi)] \sin^2(i) + \sigma_z^2 \cos^2(i) . \quad [10a]$$

Applying [10a] with $\varphi = 0$, we can calculate the tangential velocity dispersion along the major axis:

$$\sigma_{\text{major}}^2 = \sigma_\varphi^2 \sin^2(i) + \sigma_z^2 \cos^2(i) , \quad [10b]$$

and the radial velocity dispersion along the minor axis, where $\varphi = \pi/2$:

$$\sigma_{\text{min}}^2(R \cos(i)) = \sigma_R^2 \sin^2(i) + \sigma_z^2 \cos^2(i) . \quad [10c]$$

We still have too much unknowns. Therefore, to retrieve one more relation between the parameters, we can use the Oort parameters (originally written for the solar neighborhood and nearly circular orbits), and with the assumption of epicycle approximation for the orbits of the stars (see §4), we can write the relation

$$\frac{\sigma_\varphi^2}{\sigma_R^2} = \frac{-B}{A-B} \quad [11]$$

where
$$A = \frac{1}{2} \left(\frac{v_c}{R} - \frac{dv_c}{dR} \right), \quad B = -\frac{1}{2} \left(\frac{v_c}{R} + \frac{dv_c}{dR} \right), \quad [12]$$

therefore
$$\frac{\sigma_\phi^2}{\sigma_R^2} = \frac{1}{2} \left(1 + \frac{R}{v_c} \frac{dv_c}{dR} \right), \quad [13a]$$

or
$$\frac{\sigma_\phi^2}{\sigma_R^2} = \frac{1}{2} \left(1 + \frac{d \ln(v_c)}{d \ln(R)} \right). \quad [13b]$$

Moreover, to be less sensitive to data noise as (Shapiro et al., 2003) did, for the circular velocity we can adopt a parametric power law, such as $v_c = v_0 R^\alpha$ (at the radii where the disk light dominates), and from [13a] we get
$$\frac{\sigma_\phi^2}{\sigma_R^2} = \frac{1}{2} \left(1 + \frac{R}{v_0 R^\alpha} \alpha v_0 R^{\alpha-1} \right) = \frac{1}{2} (1 + \alpha)$$

The direct value of v_c , that is to say the value of v_0 with which to determine $\alpha = \frac{\ln(v_c/v_0)}{\ln R}$, can be retrieved from spectroscopic analysis of gas rotation, or from the asymmetric drift equations [9] as we will see later on.

Now we can determine the σ_z/σ_R ratio, rewriting [10b] and [10c] expressions without unknowns:

$$\begin{aligned} \sigma_{\text{major}}^2(R) &= \frac{1}{2} (1 + \alpha) \sigma_R^2 \sin^2(i) + \sigma_z^2 \cos^2(i) \\ \sigma_{\text{min}}^2(R \cos(i)) &= \sigma_R^2 \sin^2(i) + \sigma_z^2 \cos^2(i). \end{aligned}$$

If there is no gas analysis available, the asymmetric drift equation [9] can help us to determine the value of v_c ; in any case, if we had the value of circular velocity, using that equation we have the opportunity to compare and test the data coming from different sources.

The data of Gerssen and Shapiro had some noise and they couldn't get the velocity dispersion directly, therefore they adopted a fitting-approach model based on the exponential laws:

$$\begin{aligned} \sigma_R &= \sigma_{R,0} e^{-R/h} \\ \sigma_z &= \sigma_{z,0} e^{-R/h} \end{aligned}$$

where the values $\sigma_{R,0}$ and $\sigma_{z,0}$ are the central values for the radial and vertical components of the velocity dispersions, and where we consider the same scale length h (the parameter that reduces the value to the reference value 0 over e constant), which means that the ratio $\frac{\sigma_z}{\sigma_R} = \frac{\sigma_{z,0}}{\sigma_{R,0}}$ is a constant.

With this choice we can rewrite the [10b] and [10c] expressions as

$$\begin{aligned} \sigma_{\text{major}}^2 &= \frac{1}{2} (1 + \alpha) \sigma_{R,0}^2 e^{\frac{-2R}{h}} \sin^2(i) + \sigma_{z,0}^2 e^{\frac{-2R}{h}} \cos^2(i) \\ \sigma_{\text{min}}^2(R \cos(i)) &= \sigma_{R,0}^2 e^{\frac{-2R}{h}} \sin^2(i) + \sigma_{z,0}^2 e^{\frac{-2R}{h}} \cos^2(i), \end{aligned}$$

to be compared with the observed values.

Using advanced instruments such as the SparsePak¹ Integral Field Unit, some authors (Westfall et al., 2005, 2008) performed a refined decomposition (within a 40° wedge about the major axis along the desired kinematic axes), without fitted forms and error-prone interpolation between the major and minor axes, as usually done in studies based only on long-slit data. The results seem to ensure more precise analyses of the galaxies kinematics.

In their analysis they assumed that:

- the epicycle approximation holds
- the velocity ellipsoid shape and orientation is independent of z ($\partial \overline{v_R v_z} / \partial z = 0$)
- both space density ν , and σ_R have an exponential radial fall off, respectively with scale lenght of h and $2h$; that is $\nu = \nu_0 e^{-R/h}$ and $\sigma_R = \sigma_{R,0} e^{-R/2h}$; this requires the mass to follow the light, $\Sigma \propto \sigma_z^2$ where Σ is the surface density, and the velocity ellipsoid axes ratios to be constant with radius
- the circular velocity is well represented by the gaseous velocity v_g .

Therefore, comparing our formulas with theirs, the epicycle approximation [13b] becomes:

$$\frac{\sigma_\phi^2}{\sigma_R^2} = \frac{1}{2} \left(1 + \frac{d \ln(v_g)}{d \ln(R)} \right)$$

and the asymmetric drift equation [8] (using the previous expression) becomes:

$$v_g^2 - \overline{v_\phi^2} = -\sigma_R^2 \left[1 - \frac{1}{2} \left(1 + \frac{\partial \ln(v_g)}{\partial \ln(R)} \right) + \frac{\partial \ln(\nu_0 e^{-R/h} \sigma_{R,0} e^{-R/2h})}{\partial \ln(R)} \right] = \frac{1}{2} \sigma_R^2 \left[\frac{\partial \ln(v_g)}{\partial \ln(R)} + \frac{4R}{h} - 1 \right].$$

The expression [10b] becomes $\sigma_{\text{major}}^2 = \sigma_\phi^2 \sin^2(i) + (\eta \sigma_R)^2 \cos^2(i)$, where $\eta = \sigma_z / \sigma_R$ is constant with radius.

Hence σ_R is the only unknown.

Figure 3 in the nex page, from (Westfall et al., 2005), shows a comparison between data measured and calculated with long-slit and with Integrated Field Unit.

¹ SparsePak is a bundle of 82 fibers, arranged in a sparsely filled, 76×77 arcsec hexagonal grid; it pipes light from one of the Nysmith foci to the Bench Spectrograp. More details in (Verheijen et al, 2002)

Figure 3

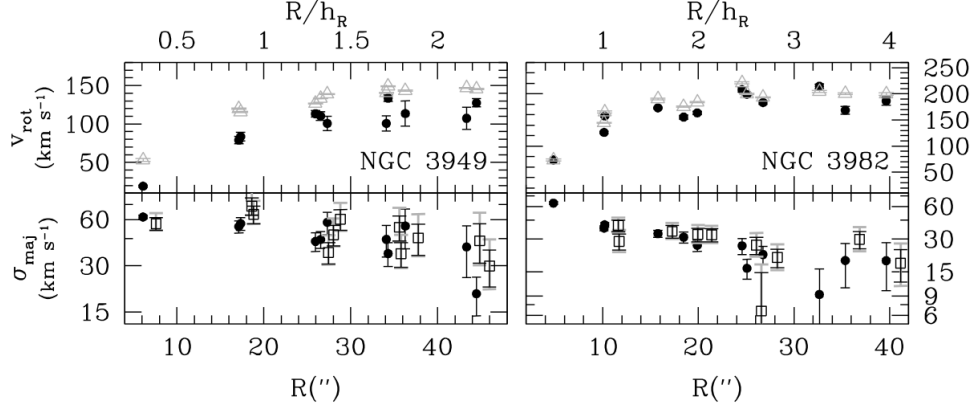


Figure 2 Comparison of measured and calculated σ_{maj} in NGC 3949 (left) and NGC 3982 (right). Top panels show the deprojected rotational velocity for both the gas (grey triangles) and stars (black points) along the major axis. Bottom panels compare the measured σ_{maj} (points) to those from our formalism using η as derived from Figure 3a (squares; offset in R for comparison). Errors on σ_{maj} are also calculated when using the 68% confidence limits for η (grey).

§4 Epicycle approximation

The assumptions of **epicycle approximation** is that, in axisymmetric potential, nearly circular orbits can be approximated as retrograde elliptical motion with epicycle frequency k on top of the circular motion of a guiding center with angular velocity Ω ; the frequency commonly reads

$$k^2 = \left(4 \Omega^2 + R \frac{d\Omega^2}{dR} \right)_{R_g}, \quad [14]$$

where R_g is the radial coordinate of the zero effective potential, and everything is defined as follows.

In a cylindrical system of coordinates (R, φ, z) , we have the following dynamic equations

$$\begin{aligned} \vec{r} &= R \hat{e}_R + z \hat{e}_z \\ \dot{\vec{r}} &= \vec{v} = \dot{R} \hat{e}_R + R \dot{\varphi} \hat{e}_\varphi + \dot{z} \hat{e}_z \\ \ddot{\vec{r}} &= \vec{a} = (\ddot{R} - R \dot{\varphi}^2) \hat{e}_R + (2 \dot{R} \dot{\varphi} + R \ddot{\varphi}) \hat{e}_\varphi + \ddot{z} \hat{e}_z \\ \nabla \Phi &\equiv \frac{\partial \Phi}{\partial R} \hat{e}_R + \frac{1}{R} \frac{\partial \Phi}{\partial \varphi} \hat{e}_\varphi + \frac{\partial \Phi}{\partial z} \hat{e}_z \end{aligned}$$

and the equations of motion, from $\vec{a} = -\nabla \Phi$, are

$$\begin{aligned} \ddot{R} - R \dot{\varphi}^2 &= - \frac{\partial \Phi}{\partial R} \\ 2 \dot{R} \dot{\varphi} - R \ddot{\varphi} &= - \frac{1}{R} \frac{\partial \Phi}{\partial \varphi} \\ \ddot{z} &= - \frac{\partial \Phi}{\partial z}. \end{aligned}$$

In the axisymmetric potential case, we have $\frac{\partial \Phi}{\partial \phi} = 0$, and calculating from $\vec{L} = \vec{r} \times \dot{\vec{r}}$ the z component of the angular momentum, $L_z = R^2 \dot{\phi}$, we see that it is constant (in fact the derivative with respect to time is zero: $dL_z/dt = 2R\dot{R}\dot{\phi} + R^2\ddot{\phi} = 0$); so the equations of motion become

$$\ddot{R} = -\frac{\partial \Phi}{\partial R} + \frac{L_z^2}{R^3} \quad [15a]$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z} \quad [15b]$$

To obtain the equations of the epicyclic motion, we consider a plane (R, z) , defined as *meridional plane*, containing the axis of symmetry of the system z and rotating with it.

Defining *effective potential* the one deriving from the energy relation (where $L_z = \dot{\phi} R^2$):

$$E = \frac{1}{2} (\dot{R}^2 + R^2 \dot{\phi}^2 + \dot{z}^2) + \Phi = \frac{1}{2} (\dot{R}^2 + \dot{z}^2) + \frac{1}{2} \frac{L_z^2}{R^2} + \Phi = \frac{1}{2} (\dot{R}^2 + \dot{z}^2) + \Phi_{\text{eff}},$$

that is where

$$\Phi_{\text{eff}} \equiv \Phi + \frac{L_z^2}{2R^2}, \quad [16]$$

we can write the motion equations as $\ddot{R} = -\frac{\partial \Phi_{\text{eff}}}{\partial R}$ and $\ddot{z} = -\frac{\partial \Phi_{\text{eff}}}{\partial z}$, in a plane rotating with $\dot{\phi} = L_z/R^2 = \Omega$. Thus in such meridional plane we can locate a region in which the effective potential has its minimum

$$0 = \frac{\partial \Phi_{\text{eff}}}{\partial R} \equiv \frac{\partial \Phi}{\partial R} - \frac{L_z^2}{R^3} \quad \text{satisfied for } R_g \text{ where } \left(\frac{\partial \Phi}{\partial R} \right)_{R_g, 0} = \frac{L_z^2}{R_g^3} = R_g \dot{\phi}^2,$$

$$0 = \frac{\partial \Phi_{\text{eff}}}{\partial z} \quad \text{satisfied anywhere in the equatorial plane } z = 0,$$

therefore the minimum potential point in the meridional plane is $(R_g, 0)$, and we expect that within this region the orbits (described by the rotation of the same meridional plane, as guiding center) are circular.

With a procedure different from that of (Chandrasekhar, 1942), who studied the question in the 1940s, a Taylor development of Φ_{eff} is carried out near this point, in which we place also the center of the orthogonal coordinate system $(x, z) = (0, 0)$, (with x aligned to R , z to z , and $x = R - R_g$), thus we obtain:

$$\begin{aligned} \Phi_{\text{eff}} = \Phi_{\text{eff}}(R_g, 0) &+ \left(\frac{\partial \Phi_{\text{eff}}}{\partial R} \right)_{R_g, 0} x + \left(\frac{\partial \Phi_{\text{eff}}}{\partial z} \right)_{R_g, 0} z + \left(\frac{\partial^2 \Phi_{\text{eff}}}{\partial R \partial z} \right)_{R_g, 0} xz + \\ &+ \frac{1}{2} \left(\frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \right)_{R_g, 0} x^2 + \frac{1}{2} \left(\frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2} \right)_{R_g, 0} z^2 + \dots \end{aligned}$$

and dropping the null derivative terms where Φ_{eff} is at minimum, the term proportional to xz because we are evaluating the symmetric Φ_{eff} around $z = 0$, and the higher order infinitesimals, we get:

$$\Phi_{\text{eff}} = \text{constant} + \frac{1}{2} \left(\frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \right)_{R_g, 0} x^2 + \frac{1}{2} \left(\frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2} \right)_{R_g, 0} z^2;$$

then, dropping the constant (subtracting an arbitrary equal value), and defining

$$k^2 \equiv \left(\frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \right)_{R_g, 0} \quad \text{epicycle frequency} \quad [17a]$$

$$v^2 \equiv \left(\frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2} \right)_{R_g, 0} \quad \text{vertical frequency} \quad [17b]$$

the equations [15a] and [15b] become the equations of harmonic oscillators

$$\ddot{x} = -k^2 x$$

$$\ddot{z} = -v^2 z$$

with solutions

$$x = X \cos(kt + \psi) \quad [18a]$$

$$z = Z \cos(vt + \zeta), \quad [18b]$$

where X, ψ , and Z, ζ are constants, and from

$$\frac{d\phi}{dt} = \frac{L_z}{R^2} = \frac{L_z}{R_g^2} \left(\frac{R}{R_g} \right)^{-2} = \frac{L_z}{R_g^2} \left(1 + \frac{R}{R_g} - 1 \right)^{-2} = \frac{L_z}{R_g^2} \left(1 + \frac{x}{R_g} \right)^{-2} \sim \Omega_g \left(1 - \frac{2x}{R_g} \right) \quad [19]$$

where Ω_g is the angular velocity at minimum of effective potential, and where we get the last term from the first two terms of a Taylor expansion for $x \ll R_g$, that is very close to the location of the effective potential minimum. Replacing for x the value [18a] and integrating, we obtain

$$\phi = \phi_0 + \Omega_g t - \frac{2 \Omega_g X}{k R_g} \sin(kt + \psi).$$

Next, from $y = R_g \phi$, we get the third coordinate of the cartesian system centered in the radial coordinate R_g of the meridional plane, that is the guiding center called **epicenter**, then a complete solution for the orbit in epicycle approximation:

$$y = -Y \sin(kt + \psi), \text{ where} \quad [18c]$$

$$Y = \frac{2 X \Omega_g}{k}. \quad [20]$$

The motion in z direction is independent from the motion in x and y , the equation [18a] and [18c] describe a retrograde elliptic motion in the xy plane (orthogonal to the meridional plane), called **epicycle**, with aspect ratio $\frac{X}{Y} = \frac{k}{2 \Omega_g}$ (from [20]) and with epicyclic frequency k .

From the [16] and [17a] definitions, where $\frac{\partial \Phi}{\partial R} = R \Omega^2$ (from the centripetal acceleration), and

$\Omega = \dot{\phi} = \frac{L_z}{R^2}$, we can derivate the [14] formula:

$$k^2 = \left(\frac{\partial (R \Omega^2)}{\partial R} \right)_{R_g} + \left(3 \frac{L_z^2}{R^4} \right)_{R_g} = \left(\Omega^2 + R \frac{d\Omega^2}{dR} \right)_{R_g} + (3 \Omega^2)_{R_g} = \left(4 \Omega^2 + R \frac{d\Omega^2}{dR} \right)_{R_g}.$$

Following (Chandrasekhar, 1942), but omitting the proof, for near circular orbits we can get the *coefficient of stability*:

$$\frac{\partial^2 \Phi}{\partial R^2} + \frac{3}{R} \frac{\partial \Phi}{\partial R} > 0, \quad \text{that is} \quad \frac{\partial}{\partial R} \left(R^3 \frac{\partial \Phi}{\partial R} \right) > 0,$$

or the theorem: a nearly circular orbit of radius R in the (x, y) plane is stable if and only if the gravitational attraction at R decreases more slowly than an inverse cube law of force.

The epicycle approximation, and the Oort parameters, can be useful in the study of the ellipsoids of velocity, whose shapes come from the ratio between the velocity dispersions along the main axes. We can rewrite the Oort parameters [12] as follows:

$$A - B = \left(\frac{v_c}{R} \right)_{R_0} = \Omega_0, \text{ then } A = \Omega_0 + B$$

$$A + B = - \left(\frac{dv_c}{dR} \right)_{R_0} = - \left(\frac{d(R\Omega)}{dR} \right)_{R_0} = - \left(\frac{d\Omega}{dR} R + \Omega \right)_{R_0},$$

then

$$A + B = 2B + \Omega_0 = - \left(\Omega + R \frac{d\Omega}{dR} \right)_{R_0},$$

and so

$$B = - \left(\Omega + \frac{1}{2} R \frac{d\Omega}{dR} \right)_{R_0}.$$

Therefore the epicycle frequency [14] can be rewritten as

$$k^2 = \left(4\Omega^2 + \frac{R 2\Omega d\Omega}{dR} \right)_{R_0} = 4\Omega_0 \left(\Omega + \frac{1}{2} R \frac{d\Omega}{dR} \right)_{R_0} = 4(A - B)(-B).$$

Considering the azimuthal velocity close to the circular velocity, so that $\overline{v_\phi} \sim v_c$, then

$$v_\phi(R_0) - v_c(R_0) = \dot{\phi} R_0 - \Omega_0 R_0 = R_0 (\dot{\phi} - \Omega_g + \Omega_g - \Omega_0)$$

and from [19], considering $\Omega_g - \Omega_0$ as part of an incremental ratio, we have

$$\begin{aligned} v_\phi(R_0) - v_c(R_0) &= R_0 \left[\Omega_g \left(1 - 2 \frac{x}{R_g} \right) - \Omega_g + \left(\frac{d\Omega}{dR} \right)_{R_0} (R_g - R_0) \right] = R_0 \left[-2 \frac{x}{R_g} \Omega_g - \left(\frac{d\Omega}{dR} \right)_{R_0} x \right] = \\ &= -2x R_0 \left[\frac{\Omega_g}{R_g} + \frac{1}{2} \left(\frac{d\Omega}{dR} \right)_{R_0} \right], \end{aligned}$$

and considering $\frac{\Omega_g}{R_g} \sim \frac{\Omega_0}{R_0}$, we get

$$v_\phi(R_0) - v_c(R_0) = -2x R_0 \left[\frac{\Omega}{R} + \frac{1}{2} \frac{d\Omega}{dR} \right]_{R_0} = -2x \left[\Omega + \frac{1}{2} R \frac{d\Omega}{dR} \right]_{R_0} = 2x B.$$

We can now calculate the azimuthal and radial velocity dispersions

$$\begin{aligned} \sigma_\phi^2 &= \overline{(v_\phi - v_c)^2} = 4x^2 B^2 = 4B^2 X^2 \overline{\cos^2(kt + \psi)} \\ \sigma_R^2 &= \overline{v_R^2} = \overline{\dot{x}^2} = k^2 X^2 \overline{\sin^2(kt + \psi)}. \end{aligned}$$

There are a lot of stars, so taking the average we map all the angles between 0 and 2π , therefore the average value of sine and cosine is the same, and in their ratio we can write

$$\frac{\sigma_\phi^2}{\sigma_R^2} = 4 x^2 B^2 = \frac{4 B^2 X^2 \overline{\cos^2(kt + \psi)}}{k^2 X^2 \overline{\sin^2(kt + \psi)}} = \frac{4 B^2}{k^2} = \frac{4 B^2}{-4(A - B)B} = \frac{-B}{(A - B)},$$

that is the [11] formula that we used in §3 to simplify the asymmetric drift equation in the anisotropic systems.

§5 Some scientific papers about disk heating

In the (Shapiro et al., 2003) paper, 6 galaxies have been measured successfully with the σ_z/σ_R ratios values that go from about 0.8 to about 0.5, with a trend (Figure 4) of the values as a function of galactic type according to the Hubble classification, while the trend to later type system remains unclear; some other authors (Kuntschner et al., 2006, Krajnović et al., 2008) investigated the velocity ellipsoid in early type galaxy, the so-called 'fast rotators', bulge-dominated galaxies, that nevertheless contain a significant disk component, finding a clear correlation between the ratio σ_z/σ_R and the galaxy B - V color as a proxy for morphological type.

Figure 4

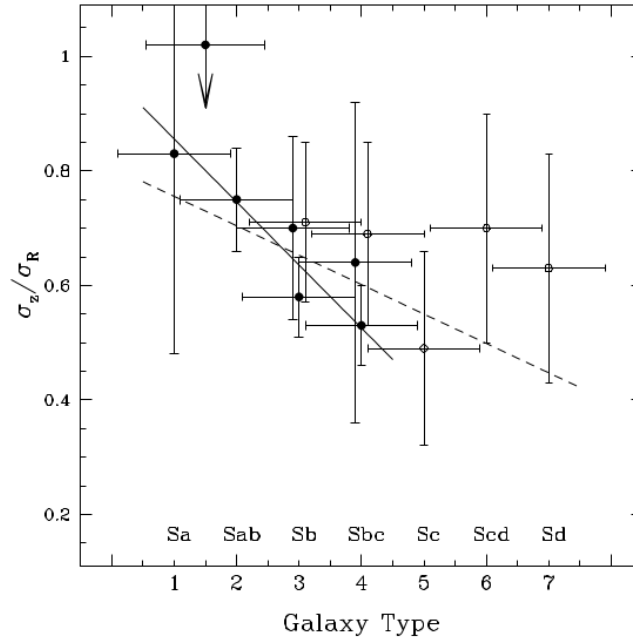


FIG. 5. — Velocity ellipsoid ratio σ_z/σ_R as a function of galactic type (Hubble stage T). The filled circles displayed here represent the results obtained kinematically for individual galaxies and are best fitted by the solid line. The open circles represent the photometric results of van der Kruit & de Grijs (1999); when combined with our results, the data are best fitted by the dashed line. Vertical error bars for our data are derived in this paper, and those for van der Kruit & de Grijs's data are statistical (the rms of determinations for multiple galaxies of the same type). Horizontal errors represent the uncertainty inherent in galaxy classification (Naim et al. 1995). The σ_z/σ_R value for NGC 2775 is an upper limit (arrow) and was not included in the linear regression analysis.

On the contrary, other authors like (Pinna et al., 2018) argue that there is no clear correlation between the shape of the velocity ellipsoid and the morphological type, and show that galaxies with the same Hubble type display a wide range of vertical-to-radial velocity dispersion ratios. The points of the 55 analyzed galaxies are distributed around a mean value, and a scatter of $\sigma_z/\sigma_R = 0.7 \pm 0.2$.

They also analyzed numerical simulations, concluding that different mechanisms might influence the shape of the velocity ellipsoid in the same way, and that the same process does not have the same impact in all the galaxies, therefore the vertical-to-radial axes ratio of the velocity ellipsoid is not a good indicator of the main source of disk heating.

Figure 5

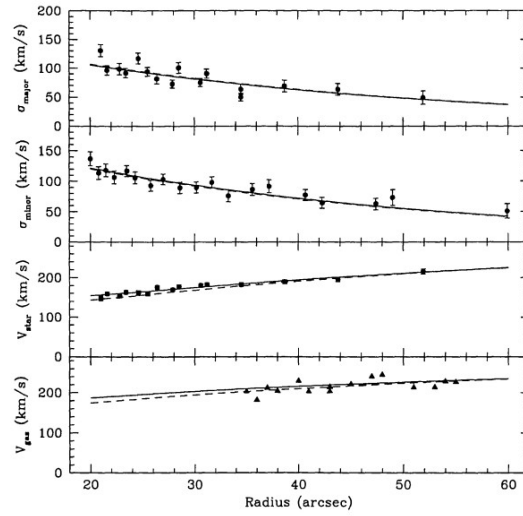


Figure 1 Plot showing the kinematic data obtained for NGC 488, and the resulting model fits. The panels show the line-of-sight velocity dispersions along the major and minor axes, and the stellar and gaseous rotation rates along the major axis.

Figure 6

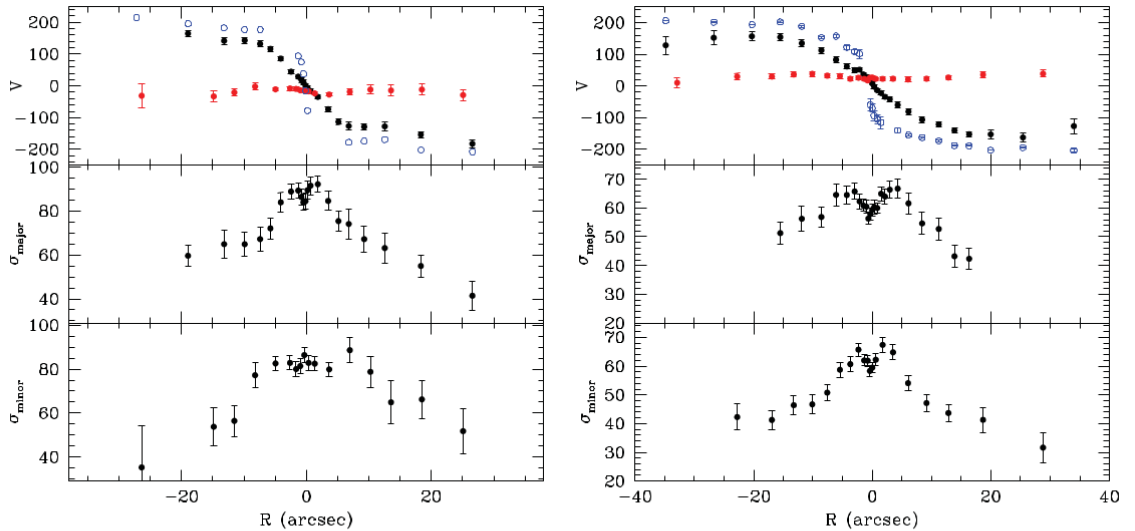


Figure 2. Observed kinematics of the two late-type spiral galaxies, left: NGC 2280 (type Scd) and right: NGC 3810 (type Sc). In all panels, the kinematic information is given in km s^{-1} . Filled circles represent major and minor axis stellar data; open circles represent gas data. Error bars for the stellar velocities are shown but are generally smaller than the plot symbols. The top panel for each galaxy shows the gas major axis rotation curve (blue open circles, corrected for inclination), the stellar major axis rotation curve (black filled circles, corrected for inclination) and the galaxy minor axis velocities overplotted (red filled circles, roughly constant at 0 km s^{-1}). The galaxy systemic velocities have been subtracted. The lower two panels for each galaxy show the major and minor axis stellar velocity dispersions, respectively. The abscissa is the radius in the disc, which, for the minor axis, was obtained through deprojection.

In Figure 5 in the previous page, from (Gerssen et al., 1997), we can see that in the inner regions of the galaxy the velocity dispersion is greater than in the outer regions, while the velocities slightly grow away from the centre; Figure 6 in the previous page, from (Gerssen and Shapiro, 2012), and Figure 7, from (Shapiro et al., 2003), show the same analysis for different galaxies.

Figure 7

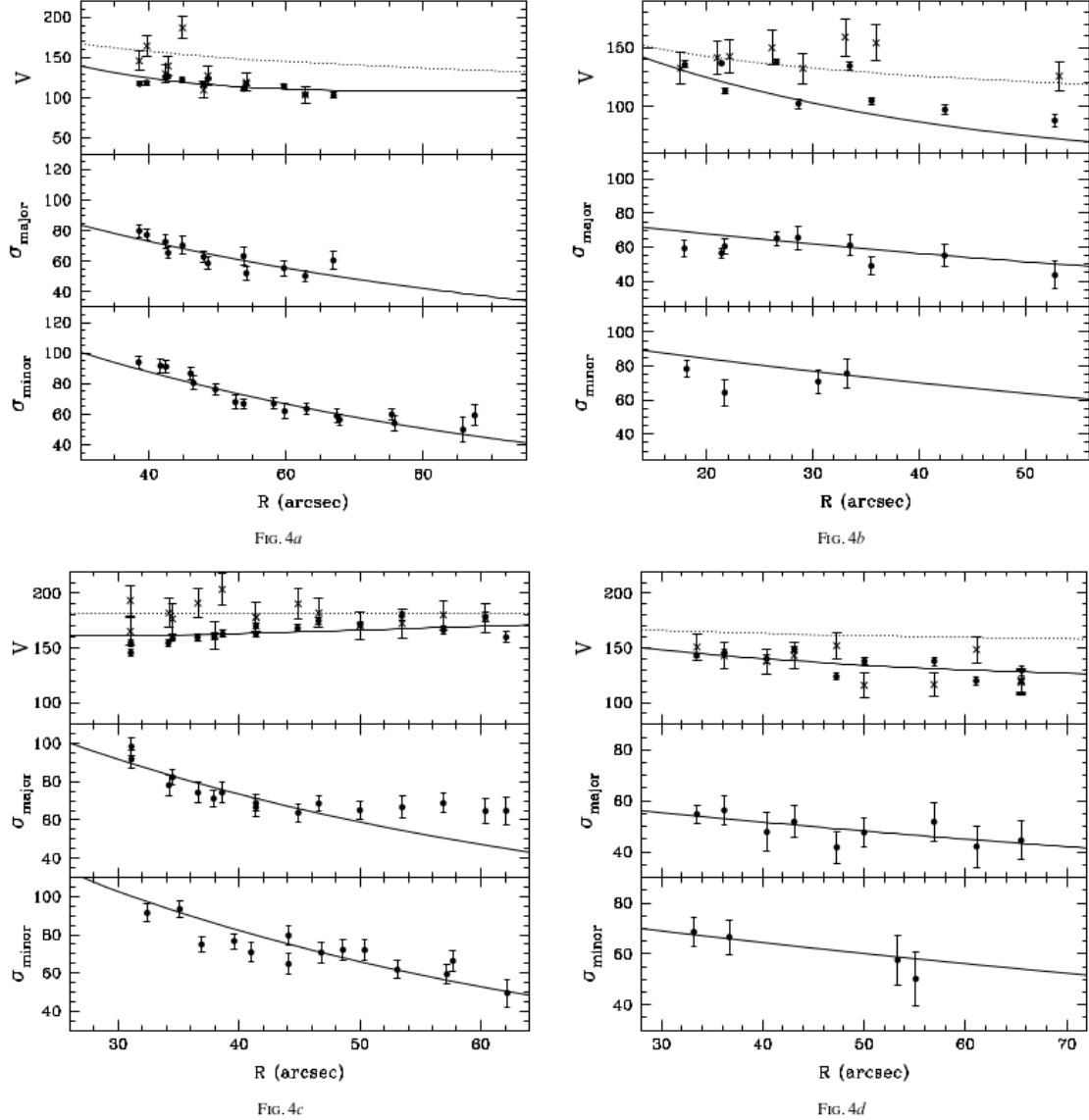


FIG. 7.—Results of fitting the model to each of the four galaxies. All kinematic quantities are in kilometers per second. Filled circles represent stellar data, and crosses represent gas kinematic data. The best-fit models for the stellar kinematics of each galaxy are shown as solid lines. The dotted lines are the circular velocities $V_c(R)$ in the models. For (a) NGC 1068, (c) NGC 2775, and (d) NGC 4030, $V_c(R)$ was predicted based on the stellar rotation curve $\bar{V}(R)$ using equation (3). For (b) NGC 2460, the gas kinematics were used to constrain $V_c(R)$.

§6 Overview on disk heating causes

Following (Gerssen and Shapiro, 2012) there are a lot of possible causes of the disk heating, basically they are the following:

- a) a star can receive a random push, a gravitational scattering, from a more massive object of the disk plane, like a giant molecular cloud (GMC, 5 to 200 pc, 10^4 to $10^7 M_\odot$)
- b) the mass enhancements associated to spiral density waves can gravitationally scatter a star
- c) the stars can be perturbed from irregular and transient spiral structure or stellar bars
- d) an external source, like a global cluster or a small satellite galaxy colliding with the disk, dissipate the energy that heats the original stellar population.

First of all we must consider that the morphologic type entails a lot of characteristics of the galaxy such as bulge to disk ratio, tightness of the spiral structure, mass surface density, gas fraction and so on, and therefore it's not easy to attribute the responsibility to one of the causes with certainty. (Gerssen and Shapiro, 2012) consider two general categories: structural properties related to the large scale evolutionary history of a galaxy, such as galaxy mass and relative prominence of the buldge, and property related to past, current and future star formation processes such as galaxy colour, tightness of the spiral structure, gas fraction. They noted that in this case the correlation with the ratio σ_z/σ_R remains, in the analysis of early type galaxies like the lenticular.

They examined the relationship of vertical and radial dispersions (σ_z and σ_R) with each other and with σ_z/σ_R , showing that σ_z/σ_R correlates strongly with σ_z but not at all with σ_R , and this suggests that kinematic observations alone can unravel the effects of disk heating mechanism that acts in three dimensions from the effects of those that act in the plane of disk.

(Gerssen and Shapiro, 2012) hypothesize two agents responsible for increasing velocity dispersion: one 'three-dimensional agent' that acts increasing both vertical and radial dispersion, and one 'radial agent' not correlated to vertical velocity dispersion. (Pinna et al., 2018) also observed that the anisotropy can be caused by a 'radial heating' as well as a 'vertical cooling'.

These agents can be respectively associated to the GMCs and to the transient spiral structure, as (Gerssen and Shapiro, 2012) analyzed in their paper; even if in the case of GMCs the correlation is not strong, and their identification as the three-dimensional heating agent is not straightforward. Differently, in the analyzed galaxies, the spiral structures correlate well with σ_R , slightly with σ_z and do not correlate with σ_z/σ_R , therefore they can be an excellent candidate for the 'radial agent'.

Let us consider in more detail the main hypothesized causes of disk heating.

Giant molecular clouds

A (young) star orbiting on the galactic plane with no vertical component of velocity, and with constant azimuthal velocity, moves in the region of the giant molecular clouds, orbiting likewise on the galactic plane, imagining with no vertical component. Nevertheless, between the star and the GMC, there is always present a differential velocity of rotation (Figure 8, from Binney and Tremaine, 2008), that is: the inner parts have greater azimuthal velocity with respect to the outer ones (the disk is assumed to have a flat circular speed curve).

As stated in (Julian and Toomre, 1966), "the radial force component somewhat resembles an impulse occurring at the instant of closest passage", and "every major gas aggregate should also serve as a catalyst for transferring angular momentum from the inner parts of a galaxy toward the outer", in this way the star could gain a small epicyclic motus that could represents a component of disk heating.

(Binney and Tremaine, 2008) studied in details the tidal effects of a relevant mass orbiting together with gas clouds; with reference to Figure 8: the cited equations are the equations of motion in the Hill's approximation (a somehow simplifying version), the Jacobi radius r_J of an orbiting stellar system represents the observational tidal radius, the maximum extent of the satellite system (with some limitations). If the initial difference in the orbital radii is $\Delta r \lesssim r_J$, the encounter reverse the direction of the orbit relative to the cloud, without any significative epicyclic motion. If $\Delta r \sim r_J$, the encounter imparts significant epicyclic motion: the epicycle amplitude is comparable with Δr and the encounter may or may not reverse the motion direction of the orbit relative to the cloud. If $\Delta r \gtrsim r_J$ the star passes the cloud and acquires a small epicyclic motion. All these epicyclic motions warm the disk.

Figure 8

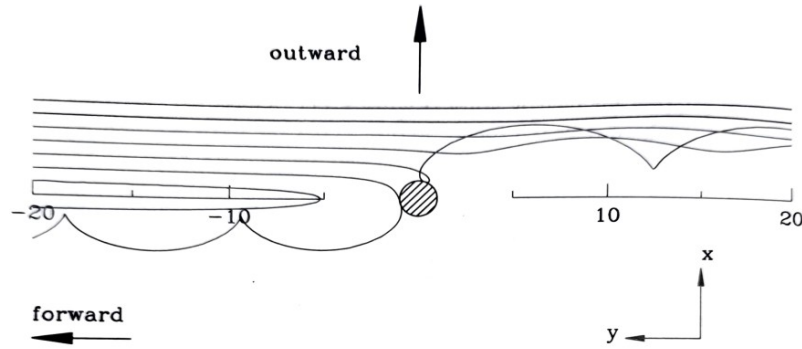


Figure 8.12 The trajectories of stars near a mass point in a disk. The orbit of the mass point is circular, as are the initial orbits of the stars. The coordinate frame co-rotates with the mass point, which is therefore fixed at the origin; in this expanded view of the area near the mass point, upwards is radially outward and the mass point is traveling to the left in an inertial frame. All orbits are restricted to the $z = 0$ plane. Circular orbits appear as straight horizontal lines. A sequence of seven orbits is shown, all initially circular with radius slightly larger than that of the point mass, so it overtakes them. The behavior of orbits inside the point mass is given by reflecting these orbits through the origin. The trajectories of the stars are described by equations (8.97). The disk is assumed to have a flat circular-speed curve, so Oort's constant $A_0 = \Omega_0/2$ and the epicycle frequency $\kappa_0 = \sqrt{2}\Omega_0$. Distances are measured in units of the Jacobi radius r_J of the mass point (eq. 8.106), which is also the radius of the circle representing its location.

Considering a different aspect of the 'impulses' coming from the galactic plane's sources, the cosmic rays, (Armillaotta et al, 2024) presented an in-depth study with many simulations on the kinematics of cosmic rays, and their pressure effect on the gases (hot and ionized) of the galactic disk, with different modes of possible transport, up to the distance of few hundred of parsecs on the z coordinate. Cosmic rays have a low number density, $3 \cdot 10^{-10} \text{ cm}^{-3}$, but a significative energy density, 1 eV cm^{-3} , that is an average kinetic energy of $1 \div 10 \text{ GeV/particle}$. But, on the basis of their simulations, it can be stated that the pressure of cosmic rays on the galactic gases determines their repositioning, but it is not sufficient to determine disk heating effects on the stars of the disk.

Spiral arms and black holes

The arms of spiral galaxies as potential causes of disk heating were studied by (Barbanis and Woltjer, 1967), but the analysis dealt mainly with motions in the galactic plane, while the effects of the arms on z axis component were negligible. A more recent study of (Jenkins, 1992), based on simulations, found that the assumed strength of spirals heating did not significantly affect and that the heating effect is probably due to both the contributions of clouds and arms, in a proportion linked by their ratio β , of quite high value ($\beta \sim 120$).

(Binney and Tremaine, 2008) analyzed the influence of spiral structures with standing wave (with fixed pattern speed) and found it negligible except in case of particular spiral shapes or recent encounters, and more that it depends on their temporal structure (duration of the transient), as considered in the *density-wave theory* which has overcome the stationary spiral structures of the Lin-Shu hypothesis. They conclude that the simplest explanation of disk heating consistent with most of the observations is the combined effects of spiral transient and molecular clouds.

(Hänninen and Flynn, 2021) have recently developed a study based on simulations, also using data from Hipparcos samples of nearby stars, and fit a heating law of the form $\sigma(t) = \sigma_0 (1 + t/\tau)^\alpha$ to the data, where σ is the stellar velocity dispersion and τ is a scale time; the observational data limit α to values between about 0.3 and 0.6 (lower than earlier studies have found), while the ratio between the velocity dispersions is $\sigma_z/\sigma_R \approx 0.5 \pm 0.1$. They simulated disk heating by modelling a patch of the galactic disk populated with tracer stars, and examined the effect of GMCs travelling on disk orbits through the patch, and of massive black holes travelling through on plunging orbits.

The ratio σ_z/σ_R of heating in the vertical and horizontal directions in the disk has been found to depend on the adopted GMC density, but this value of $\Sigma \sim 5 M_\odot/\text{pc}^2$ for the solar neighborhood does not heat the disk enough. They also run simulations of the local disk heating being caused by massive black holes ($10^6 \div 10^7 M_\odot$) of the dark halo penetrating the disk and perturbing the stellar orbits. The effect is not that significant, but together with the GMC then the heating would also push the ratio of the velocities dispersion higher to $\sigma_z/\sigma_R \approx 0.63$, barely within the observational limits. The authors emphasized that a new study with the contribution of spiral arms is appropriate and planned to investigate further.

The simulations made by (Sellwood and Preto, 2002) showed that spiral waves of moderate amplitude cause substantial changes to the angular momentum of many stars in the galactic disk, and stars near to corotation are affected most strongly: for a wave of $\sim 20\%$ overdensity, the rms (rotation measures) change for particles near to corotation is $\sim 18\%$ of their initial angular momenta. Yet these changes produce no increase in random motion near corotation, while the same wave causes mild heating near its Lindblad resonances, the orbital resonance in which an object's epicyclic frequency is a simple multiple of the forcing frequency of the spiral wave.

Interestingly, the stirring of the disk by spiral waves has implications for the chemical abundance gradient: if stars can migrate considerably from their birthplace, the abundance gradients when they formed will be diluted. For example it's possible that the Sun migrated 1.9 kpc from its birthplace, as claimed by (Wielen et al., 1996), who based their prediction on a diffusion coefficient empirically determined by fitting an observed age-velocity relation, given a significative radial gradient for stars metallicity in the galaxy.

Collisions with satellite galaxies

The collisions between galaxies of similar masses usually produce a new object different from the original ones, but collisions with small satellite galaxies can grow the disk mass, storing kinetic energy that means more random velocity of the stars.

(Schwarzkopf and Dettmar, 2000) made a statistical study on the effects of tidal interactions and minor merges (on the range $M_{\text{sat}}/M_{\text{disk}} \approx 0.1$) on the radial and vertical disk structure in a sample of 110 highly inclined / edge on spiral galaxies. The main conclusions are the following:

- the structural changes of galactic disks affected by low-mass satellite are most noticeable in the direction perpendicular to the disk plane
- while the majority of non-interacting galaxies posses a typical exponential disk scale height of $z_0 \approx 700$ pc, disks of minor merges were found systematically thicker with $z_0 \approx 1.3$ Kpc
- the ratio of radial to vertical scale parameters, i.e. the normalized disk tickness h/z_0 , is ≈ 1.7 times smaller for the merging sample
- ratios $h/z_0 > 7$, as they are typical for flat galaxies, are completely missing in the merging sample; this implies that vertical disk heating is most efficient for such thin disks
- the radial disk structure of merging galaxies shows no statistically significant changes.

(Velázquez and White, 1999) performed a set of N-body simulations to study the interaction between disk galaxies and merging satellites, with the aim of determining the disk kinematical changes. The damaging effects (heating and thickening) on the disk are very different for satellites on prograde and retrograde orbits: a satellite on a prograde orbit tends to heat the stellar disk while its retrograde counterpart induces a coherent response of the disk in the form of a tilt; a massive satellite on a retrograde orbit may be accreted by a spiral galaxy without destroying its disk. Furthermore, a massive central bulge may reduce the vertical heating of the disk for prograde orbits (but weaker for retrograde).

Perhaps we can conclude that collisions with satellite galaxies appear to be the main responsible for the heating of the disk but more in-depth studies are certainly necessary to better understand the dynamics of this process.

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