

The URD Framework: Cosmological Distances from Redshift in a Non-Standard Phenomenological Framework

Andrea Brussi*

July 7, 2026

Abstract

A phenomenological framework for determining cosmological distances is proposed, based on a non-expanding spacetime interpretation consistent with current observational datasets. The model introduces a redshift mechanism associated with cumulative photon energy attenuation along the line of sight, formulated in a way that is not equivalent to standard tired light scenarios. The approach is constructed to remain compatible with established relativistic principles whereas adopting boundary conditions distinct from the metric expansion paradigm. Distance estimation is achieved through the URD equation (U energy density, Redshift, Distance), which incorporates two phenomenological parameters that describe conformal geometric dissipation in extended gravity and an effective relativistic path-length correction. Within this formulation, the model addresses the primary shortcomings traditionally associated with energy dissipation effects, specifically reconciling time dilation in supernova light curves and the Tolman surface brightness relation. The model is calibrated using the Pantheon+ supernova sample by comparing and cross-validating distances inferred from the proposed redshift mapping with redshift-independent indicators, with additional comparison performed against baryon acoustic oscillations, radio galaxy samples, and high-energy quasars. Across these datasets, the framework reproduces observational constraints over a wide redshift range within a phenomenological, non-standard cosmological setting. The results indicate that alternative, non-expansion-based mappings between redshift and distance can be constructed that remain consistent within the observational range considered, suggesting a phenomenological alternative worth further study.

Keywords: cosmological distance ladder; redshift-distance relation; non-expanding spacetime; cosmic microwave background; extended gravity; Pantheon+ sample; baryon acoustic oscillations; Tolman test; time dilation; geodetic correction

Premises and working assumptions for the URD Framework

Recent developments in observational cosmology have renewed interest in the extent to which standard distance–redshift interpretations face the phenomenology of high-redshift data. In particular, measurements from James Webb Space Telescope (JWST; see details in Gardner 2006 [1]), see e.g. McGaugh et al. 2024 [2], Wright 2024 [3], di Valentino et al. 2025 [4] (JWST anomalies section), together with ongoing discussions on BAO (DESI Collaboration¹ 2024 [5]), the Tolman surface-brightness test (Lovyagin et al. 2022² [6], Lerner et al. 2014 [7], Lerner 2018 [8]), have highlighted both the strengths of the standard cosmological framework and the

*Independent Researcher, Treviso, Italy. ORCID: 0009-0000-3924-1563. Email: info@andreabrussi.it

¹DESI (Dark Energy Spectroscopic Instrument) Collaboration is an international scientific collaboration dedicated to the study of dark energy, comprising over 450 researchers from more than 70 institutions around the world, and relies on observations made using a specially designed spectrograph.

²A recent study not widely accepted.

presence of open interpretative issues in its observational reading: e.g. the recent study Conselice et al. 2026 [9] (preprint), using the high-quality surface brightness (SB) data Beltrán et al. 2024 [10], must resort to significant evolutionary corrections to force-fit these observations into the standard CDM model. The pursuit of alternative theoretical frameworks is further motivated by comprehensive recent reviews such as those by Abdalla et al. 2022 [11], which systematically document the growing tensions and anomalies within the standard model. Their work underscores the recognized need in the field to explore new avenues to resolve the current observational impasse.

Some recent studies have attempted to address these critical issues by comparing the theoretical behavior of the standard model with new conjectures, or by proposing non-expansive models featuring novel interpretations of the cosmological redshift. For example, Shamir 2024 [12] suggests a correction to redshift measurements related to the rotation mode of galaxies, which appears to reproduce the phenomenology of Zwicky's tired-light model [13]. Tremblin 2024 [14] theoretically analyzes local volumetric expansion factors and background expansion as 'integrators' of the cosmological principle of expansion (maintaining a dynamic framework). Cody 2025 [15] (preprint) offers a methodological contribution adopting a non-expansive model and a 'cumulative motion kernel', defined from observational invariants, which represents a scaling factor analogous to the function of $a(t)$ in the Friedmann–Lemaître–Robertson–Walker model (FLRW; Friedmann 1922 [16], Lemaître 1927 [17], Robertson 1935 [18], Walker 1936 [19]). Finally, Booth 2024 [20] (preprint) proposes a theoretical mechanism based on the contraction of atomic matter (Jeans' contraction) as the source of cosmological redshift, though without direct comparison with observational data.

In this research context that attempts to address the difficulties of the standard model, it is useful to explore whether the observed redshift–distance relations can also be described within alternative phenomenological frameworks. The present work is intended as an original contribution in this direction: it develops a non-standard redshift-based distance framework and examines its consistency with the observational relations considered. These assumptions do not constitute a complete cosmological model, but rather provide a minimal interpretive structure within which the observational relations explored in this work with very large samples of data can be consistently analyzed.

The core framework of this study consists of the following Sections, which are fully developed in the subsequent text.

- 1 The universe is assumed to be spatially infinite; consequently, the anthropically knowable part can be considered infinitesimal.
- 2 On large scales, the observable universe is treated as effectively isotropic, flat, and static. While General Relativity (GR) describes local curvature due to mass-energy concentrations, we explore a boundary condition where these local perturbations do not result in a global metric evolution (expansion or contraction), effectively maintaining a quasi-Euclidean global geometry.
- 3 Net of proper motions, the redshift of remote sources is assumed to be a wavelength-independent process (consistent with observations) that does not arise from a Doppler effect, as a non-expanding geometry is adopted.
- 4 The Cosmic Microwave Radiation (CMR) is treated as an isotropic and uniform radiation field pervading the observable universe, whose spectrum is empirically described with high precision by a Planckian distribution.
- 5 The CMR is modeled as the thermal radiation emitted by the universe acting as a black body in thermodynamic equilibrium (according to Kirchhoff's law), represented by the energy density U^* .

- 6 The propagation of incident photons is modeled as a dissipative process within an *extended gravity* framework. They undergo a cumulative conformal energy attenuation driven by the isotropic background scalar curvature of spacetime, which is physically sourced by and in thermodynamic equilibrium with the U^* radiation field.
- 7 The observed redshift is interpreted as a manifestation of this conformal geometric dissipation during propagation, representing a progressive attenuation of the photon energy within a static spacetime rather than a metric stretching of space.
- 8 The energy loss is proportional to the distance traveled by the photons propagating through the spacetime curved by the U^* energy density. This geometric propagation process leads to a distance relation that incorporates a power law associated with the geodesic path excess, scaled by the empirical attenuation parameter. Some known critical issues historically associated with the classical interpretation of photon energy loss known as *tired light*—conceptually distinct from our framework—were analyzed and shown to support the URD model, rather than presenting difficulties. In particular, time dilation for the Sn Ia light curves (see §8.3.1) and the Tolman surface brightness test (see §8.3.2–§8.3.4) are naturally and accurately reproduced. The power law must be completed with a factor dependent on time dilation and becomes Eq. (15), with the definition Eq. (1):

$$D_{\text{URD}} = \frac{\xi}{\delta_g \alpha_{U^*}} (\xi^{\delta_g} - 1) \quad [\text{Mpc}]$$

$$\xi = \frac{\lambda_o}{\lambda_e} = 1 + z \geq 1 ,$$

where D_{URD} represents the estimate of the source distance in Mpc, determined based on the URD framework, from which the radiation emitted at wavelength λ_e is observed from wavelength λ_o ; the parameter α_{U^*} ($[\text{Mpc}^{-1}]$) is the attenuation coefficient (see §8.1), an empirical parameter associated with the energy density U^* ; δ_g is an empirical fine-tuning parameter associated with the geodesic excess (see §8.2); z is the conventional redshift.

- 9 We obtain the phenomenological parameters and calibrate the model with the Pantheon + dataset (1,543 unique SnIa, see §9.1), comparing the distances determined using the redshift-independent method to analyze the luminosity curves of the same SnIa. The results were statistically analyzed in depth (see §9.3), including cross-validation (see §9.2) to test a Pantheon+ sub-dataset and the predictive capabilities of the suggested equation.
- 10 The proposed cosmological Eq. (15) and the phenomenological parameters Eq. (24) were then tested by comparison with BAO (Baryon Acoustic Oscillations) analysis (see §10.1), and with some other large dataset of 10,888 radio galaxies (see §10.2) and 2,421 quasars (see §10.3), confirming that the URD model consistently determines reliable distances to distant sources, compared with many independent redshift calculation methods and across the entire electromagnetic spectrum.
- 11 Conclusions
- 12 Appendix A, About the CMR conjecture on Cosmic Microwave Radiation
- 13 Appendix B, A framework for a globally static cosmology

1 Spatially infinite universe

Assuming a spatially infinite universe provides a consistent working framework and avoids ambiguities associated with boundary conditions. In this context, the analysis is restricted to the

observable domain (see footnote 25), without attempting to infer global properties beyond what can be empirically constrained. The present framework does not rely on specific assumptions about the global origin or thermodynamic state of the universe, and is compatible with scenarios involving a low-entropy past and subsequent evolution, without requiring a specific initial condition.

Indirectly, this framework may also provide an alternative perspective on the anomaly, cited in the premise, such as the apparent presence of evolved galaxy populations at high redshift. Within the present interpretation, such observations can be accommodated without requiring accelerated early structure formation, allowing for the existence of both disk and elliptical galaxies through standard evolutionary pathways, with or without mergers. In this context, the apparent tension with the timescales inferred in the standard cosmological model is not present.

A conjecture for a spatially infinite, non-time-limited, globally static cosmology framework is briefly developed in Appendix B (§13).

Consequently, within the scope of the URD framework as a ‘test framework’ for the consistency of the cosmological distance equation and its phenomenological test, the supported hypotheses of isotropy, non-expansion, and energy conservation concern the anthropically knowable local context and cannot claim infinite universality. That would be only an unprovable philosophical speculation. Furthermore, given that the assumed initial conditions are physically *permissible*, their specific statistical weighting becomes secondary within an infinite spatial and temporal context.

2 Large scale Universe

As stated, it is believed that in the knowable universe all physical laws determined ‘locally’ so far are valid (also with the interpretive support of the *entity time*). Observations confirm that, on a large scale, this universe can be considered homogeneous and isotropic. The space metric is considered on average Euclidean, without curvature except locally due to the ‘concentrated’ presence of mass-energy (consistent with Einstein’s GR; Einstein 1916 [21]) and the laws of thermodynamics are valid.

In this context, we developed a phenomenological study based on the well-known cosmological observations, redshift and CMR, which are considered fundamental for the interpretation of observable Nature.

3 Redshift

The redshift is defined as the increase in wavelengths observed in the spectra of remote sources, defined by the relation also reported in the premise:

$$\xi = \frac{\lambda_o}{\lambda_e} = 1 + z \geq 1 . \quad (1)$$

It is always observed achromaticity, that is

- the ξ ratio is the same for all wavelengths of radiation emitted by a remote source.

In the absence of expansion of the universe and therefore of recession of sources, the observed redshift (net of the proper motions of the source and observer) can no longer be attributed to a Doppler effect (Doppler 1842 [22]). The following interpretations and conjectures have been developed to derive and empirically test a distance equation within this theoretical framework.

4 CMR (Cosmic Microwave Radiation)

The microwave radiation discovered by Penzias and Wilson 1965 [23], commonly called CMB, is highly isotropic and uniform, has a density of 410 photons per cm^3 (see, e.g., Muller et al.

2013 [24]) and is described by a spectrum reproduced by a Planck's distribution (Planck 1900 [25]) with enormous precision (the value of the root mean square deviations, rms, is estimated to be 50 parts per million of the peak of the spectral distribution, see Fixsen et al. 1996 [26]), with its maximum at wavelength $\lambda_{\text{CMR}} = 1.9 \cdot 10^{-3}$ [m]. The small anisotropies and the acoustic peaks observed in the radiation, net of the dipole effect due to the motion of our solar system with respect to the rest frame of the CMR (a Doppler effect), can be justified by the nature of this emission, analyzed in the CMR conjecture detailed in Appendix A (§12).

5 Universe as a black body

In Appendix A (§12) the CMR conjecture describes how the universe can be considered a blackbody that, based on Kirchhoff's law (Kirchhoff 1860 [27]), emits radiation that constitutes an energy density. In the URD framework, this radiation is called U^* and is represented by the CMR, which is treated as a rarefied photon gas in which the photons behave similarly to a perfect gas (with non-interacting atoms), for which are applicable Bose-Einstein statistics (Bose & Einstein 1924 [28]) with zero chemical potential. As is known, the spectral energy density per unit volume according to Planck's distribution (Planck 1900 [25]), in SI units, is:

$$u_\nu(T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/(k_B T)} - 1} \quad [\text{J m}^{-3} \text{Hz}^{-1}] . \quad (2)$$

6 Extended gravity foundations of the URD dissipative mechanism

The physical mechanism driving the progressive energy attenuation αU^* of propagating photons (see §8.1) is formulated within the framework of Extended Theories of Gravity (ETGs). This class of theories was pioneered by Starobinsky (see Starobinsky 1980 [29]) to describe singularity-free cosmological scenarios, and extensively developed by Capozziello (see Capozziello 2002 [30], Capozziello et al. 2003 [31]) to model curvature as a dynamical source of cosmic energy flows (a paradigm known as *curvature quintessence*). In these theories, the standard Einstein-Hilbert action is generalized by replacing the linear Ricci scalar curvature R in the gravitational Lagrangian with a non-linear function, denoted $f(R)$. Within this proposed framework, the cosmological redshift is interpreted as a purely geometric phenomenon arising from wave propagation through a spacetime characterized by a non-zero background curvature R_{bkg} .

In standard General Relativity, the electromagnetic Lagrangian is conformally invariant, meaning that classical photons propagate along null geodesics without experiencing any intrinsic energy loss. However, as demonstrated in the comprehensive review by Capozziello & de Laurentis 2011 [32], the introduction of higher-order terms in the modified action naturally breaks this conformal invariance. Broken symmetry induces a non-minimal coupling between the electromagnetic vector potential A^μ and the background scalar curvature R_{bkg} .

Specifically, this non-minimal geometric coupling modifies the standard covariant Maxwell equations. In the Lorenz gauge ($\nabla_\mu A^\mu = 0$), the electromagnetic wave equation in a homogeneous static background space acquires a first-derivative geometric damping term:

$$\square A^\mu + \gamma R_{\text{bkg}} \partial_t A^\mu = 0 , \quad (3)$$

where $\square \equiv \partial_t^2 - \nabla^2$ is the d'Alembertian operator and γ is a dimensionless conformal coupling constant.

The term $\gamma R_{\text{bkg}} \partial_t A^\mu$ mathematically acts as a geometric friction (or conformal drag). It represents a continuous, non-conservative transfer of energy from the propagating wave to the background geometry, yielding the progressive amplitude attenuation.

Under the working assumption of global thermodynamic equilibrium (see §5), the isotropic background scalar curvature R_{bkg} of this static universe is physically sourced by and in equilibrium

with the diffuse Cosmic Microwave Radiation (CMR) energy density U^* . Thus, the scalar curvature and the radiation energy density are coupled such that $R_{\text{bkg}} \propto U^*$. The empirical attenuation parameter α_{U^*} is therefore directly proportional to this background curvature:

$$\alpha_{U^*} = \frac{1}{2}\gamma R_{\text{bkg}} \propto U^* , \quad (4)$$

reflecting the fundamental coupling between the photon's propagation and the global energy-density-induced geometry of the vacuum.

This purely geometric mechanism naturally resolves two of the physical challenges historically associated with the classical *tired light* hypothesis, specifically concerning achromaticity and image blurring. First, because the background curvature R_{bkg} is a universal geometric property of the metric, the conformal dissipation is intrinsically achromatic. It affects all electromagnetic wavelengths identically—from γ -rays bands to the 21 cm radiation of neutral hydrogen. Second, because the attenuation is a continuous wave-propagation effect along smooth null geodesics rather than a discrete scattering process, photon directionality and momentum are perfectly preserved. This guarantees the absolute sharpness of images of distant sources, completely avoiding any observational image blurring.

7 Redshift and energy loss of incident photons

Within this geometric framework, the continuous conformal dissipation of electromagnetic energy directly manifests itself as the observed cosmological redshift. As an electromagnetic wave propagates, its wavelength increases whereas its frequency and energy decrease. The total energy attenuation of a photon along its path, based on Eq. (1), is described by:

$$\Delta E = E_o - E_e = \frac{hc}{\lambda_e} \left(\frac{1}{\xi} - 1 \right) \leq 0 . \quad (5)$$

To preserve global energy conservation within a static spacetime, this dissipated energy is assumed to be transferred directly to the physical quantum vacuum. Within this steady-state thermodynamic framework, the CMR (U^*) can be phenomenologically interpreted not as the relic of a primordial singularity, but rather as the cumulative, thermalized energy deposit of propagating radiation, continuously re-equilibrated by the vacuum background. This self-regulating thermodynamic process is hypothesized to maintain the Planckian spectrum of the U^* field, whereas establishing a physical, energy-dissipative horizon for remote observations (see footnote 25).

8 URD distance equation and analysis of the main critical issues

In this Section, we derive the URD distance equation for the distance and analyze the main critical issues associated with the incident photon energy dissipation model.

8.1 Determination of the URD framework distance equation

Within the proposed extended gravity framework, the propagation of monochromatic electromagnetic radiation from a remote source experiences a continuous conformal dissipation. This geometric attenuation is governed by the background scalar curvature R_{bkg} , which we consider physically determined by the energy density of Cosmic Microwave Radiation U^* . Consequently, the variation dI_S of the incident energy flux along the infinitesimal radial coordinate dr represents a differential geometric attenuation described by:

$$\frac{dI_S}{dr} = -\alpha_{U^*} I_S , \quad (6)$$

where $\alpha_{U^*} > 0$ ($[\text{m}^{-1}]$, expressed for convenience in $[\text{Mpc}^{-1}]$) defines the *geometric attenuation coefficient* linked to the background curvature sourced by the energy density U^* , and the *straight radial distance* r (the geometrical distance in the flat grid of coordinates of the universe) is *not* the *actual geodesic distance*³ l traveled by the photon (see Eq. (11) and in §8.2 the physical interpretation of their difference). The parameter α_{U^*} is hypothesized (based on the observed achromaticity of the redshift) to be independent of the wavelength of the radiation, representing the continuous loss of radiation intensity per unit coordinate length due to the metric-driven conformal coupling.

For the evaluation of this energy loss, the observed radiation flux can be expressed in terms of energy intensity as:

$$I_\lambda = \frac{N \Delta t h c}{A \lambda} , \quad (7)$$

where $N \Delta t / A$ is the total number of photons in the time interval Δt in the area A (which remains constant, in the absence of expansion). Eq. (6) can be rewritten as:

$$\frac{dI_S}{I_S} = -\alpha_{U^*} dr , \quad (8)$$

which has separable variables and can therefore be integrated from emitted radiation $I_{\lambda_e S}$ to observed radiation $I_{\lambda_o S}$. In particular, simplifying the common factors in the ratio and considering the redshift as in Eq. (1), we have the following.

$$\int_{I_{\lambda_e S}}^{I_{\lambda_o S}} \frac{dI'}{I'} = \ln \frac{I_{\lambda_o S}}{I_{\lambda_e S}} = \ln \frac{1}{\xi} . \quad (9)$$

Therefore, from Eq. (8) and Eq. (9) we can consider the differential energy loss along the path:

$$d(\ln \xi) = \alpha_{U^*} dr . \quad (10)$$

We must then consider the actual path l relates to the radial distance r through a correction factor $\delta_g > 0$, representing the excess further explored in §8.2, hence we can write: $dl = \xi^{\delta_g} dr$, or

$$dr = \xi^{-\delta_g} dl , \quad (11)$$

and substituting Eq. (11) in Eq.(10):

$$\frac{d\xi}{\xi} = \alpha_{U^*} \xi^{-\delta_g} dl . \quad (12)$$

Isolating dl and integrating from the source ($l = 0$, $\xi = 1$) to the observer ($l = \tilde{D}_{\text{URD}}$, $\xi = \xi_{\text{obs}}$), from Eq. (12) we get the following.

$$\int_0^{\tilde{D}_{\text{URD}}} dl = \frac{1}{\alpha_{U^*}} \int_1^{\xi_{\text{obs}}} \xi'^{\delta_g-1} d\xi' , \quad (13)$$

and finally from Eq. (13) we get:

$$\tilde{D}_{\text{URD}} = \frac{1}{\delta_g \alpha_{U^*}} \left(\xi_{\text{obs}}^{\delta_g} - 1 \right) \quad [\text{Mpc}] , \quad (14)$$

where $\tilde{D}_{\text{URD}}$ represents the estimate of the source distance in Mpc, determined on the basis of energy loss along the actual path, a first evaluation of the attenuation of monochromatic radiation.

³We consider that both dl and dr are positive as the photon approaches the observer.

The Eq. (14) must be completed by multiplying it by the factor $(1+z) = \xi$ due to the time dilation associated with energy loss (see §8.3.1 for a physical explanation); therefore, the source distance based on the URD framework is:

$$D_{\text{URD}} = \frac{\xi}{\delta_g \alpha_{U^*}} \left(\xi^{\delta_g} - 1 \right) \quad [\text{Mpc}] , \quad (15)$$

which is valid for the entire spectrum of the source.

Since $\lambda_o \geq \lambda_e$ and $\xi \geq 1$, and the parameters are positive, the value D_{URD} resulting from Eq. (15) is always correctly positive, and in the absence of redshift (when $\lambda_o = \lambda_e$ and $\xi = 1$) the distance is coherently zero. We can also control the trend of the equation's gradient when $\xi \rightarrow 1^+$:

$$\lim_{\xi \rightarrow 1^+} \frac{dD_{\text{URD}}}{d\xi} = \lim_{\xi \rightarrow 1^+} \frac{(\delta_g + 1) \xi^{\delta_g} - 1}{\alpha_{U^*} \delta_g} = \frac{1}{\alpha_{U^*}} , \quad (16)$$

a finite value related to the Hubble constant (see Eq. (29)), that is, the same relation of the Hubble–Lemaître law Eq. (25) for low redshift (or $\xi \rightarrow 1^+$). It can be seen that the concavity of the curve Eq. (15) depends on the sign of the factor $(\delta_g + 1)$, therefore the curve is always upward-concave.

8.2 A physical analysis of the geodesic correction factor

In a static, non-expanding cosmology, the observed dimming of high-redshift sources requires a geometric or physical correction to the idealized Euclidean distance. The parameter δ_g accounts for a cumulative increase in the effective geodesic path length due to local gravitational perturbations (such as lensing and local potentials). Within the adopted $f(R)$ framework, the non-vanishing background scalar curvature R_{bkg} implies that spacetime is globally quasi-flat; consequently, photon geodesics deviate statistically from Euclidean straight lines, accumulating a path-length excess over cosmological distances. Because this excess depends on the distribution of matter and local potentials along the line of sight, δ_g is treated as an empirical parameter constrained by the data.

To test the stability of this geometric interpretation, a dedicated subsample analysis was performed on the Pantheon+ database (see §9). The parameter δ_g was fitted independently across six nested subsamples defined by progressively larger maximum redshifts z_{max} , whereas holding the global attenuation parameter α_{U^*} fixed at its optimised value. For each subsample, δ_g was determined by bounded minimisation over the interval $[-0.25, 0.75]$, with 1σ uncertainties estimated via bootstrap resampling (500 iterations).

The results are presented in Table 1 (see also Figure 1 for the corresponding trend). At $z \leq 0.01$, the fit is unstable and hits the grid boundary, which is physically expected due to the dominance of local peculiar velocities. From $z \geq 0.05$ onward, δ_g quickly stabilises within a narrow range (0.22–0.29) with rapidly decreasing uncertainties, converging smoothly to the full-sample value of 0.260 ± 0.013 . This stability across more than a decade in redshift ($0.05 \leq z \leq 2.26$) confirms that δ_g captures a robust, scale-independent geometric property of the background rather than a local fitting artefact.

Finally, we note that the two parameters α_{U^*} (governing the global energy attenuation) and δ_g (modulating the geodesic path excess) represent complementary aspects of photon propagation in a quasi-flat $f(R)$ metric. Their empirically determined ratio, $\delta_g/\alpha_{U^*} \sim 10^3$ Mpc, defines a characteristic cosmological length scale of approximately 1 Gpc, suggesting that the conformal geometric dissipation becomes significant only over large cosmological baselines. A formal first-principles derivation of this relationship is deferred to future work.

Table 1: Fitted values of δ_g on nested Pantheon+ subsamples. σ is estimated via bootstrap resampling (500 iterations). The $z \leq 0.01$ subsample is dominated by peculiar velocities and the optimiser hits the grid boundary; that result is reported for completeness only.

Subsample	N	δ_g	$\sigma(\delta_g)$	χ^2_{red}
$z \leq 0.01$	113	-0.250^*	0.384	0.925
$z \leq 0.05$	648	0.269	0.205	0.577
$z \leq 0.10$	741	0.226	0.145	0.560
$z \leq 0.20$	948	0.251	0.054	0.544
$z \leq 0.50$	1491	0.299	0.017	0.498
Full sample	1701	0.260	0.013	0.478

* Boundary value; fit unstable due to peculiar velocity dominance.

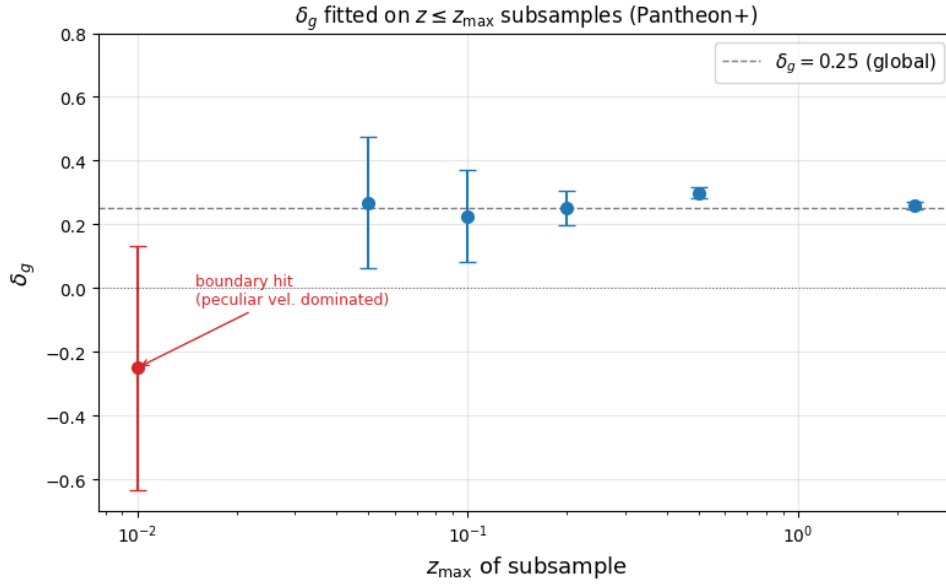


Figure 1: Fitted values of δ_g on nested Pantheon+ subsamples reported in Table 1.

8.2.1 Conclusion on dark energy equivalency

In the standard Λ CDM model, the observed dimming of high-redshift supernovae is attributed to an accelerated expansion of space, traditionally parameterized by the cosmological constant Λ or a dark energy density component (Ω_Λ), see Riess et al. 1998 [33] and Perlmutter 1999 [34]. This parameterization effectively increases the distances inferred from remote sources, accounting for their observed brightness.

Conversely, the URD framework interpretation maintains that the sources remain at their static coordinates. In this model, the observed dimming is not a symptom of accelerating space, but it is due to a geodesic correction in the photon's path, accounting for the very same observational discrepancy that Ω_Λ seeks to explain. However, whilst Ω_Λ assumes the existence of a repulsive force parameterized by the cosmological constant, the parameter δ_g accounts for local gravitational distortions along the path, in accordance with GR without Λ correction (see Appendix B §13 for explanations on the assumed physical context).

8.2.2 Analytical analysis

To analyze the trend of Eq. (15) relation with Eq. (24) parameters (obtained from calibration with Pantheon+ sample in §9), we examine its second-order derivative with respect to the independent variable ξ ; it represents the marginal rate of distance change with respect to ξ , given by:

$$\frac{d^2 D_{\text{URD}}}{d\xi^2} = \frac{(\delta_g + 1) \xi^{\delta_g - 1}}{\alpha_{U^*}}.$$

Under conditions $\alpha_{U^*} > 0$ and $\delta_g > 0$ (see Eq. (24)), the second derivative remains strictly positive for all $\xi > 1$ (always verified, see Eq. (1)). This implies that the function is strictly convex, exhibiting a monotonic ‘acceleration’ in distance as ξ increases. Physically, this suggests that the model describes a system where the dimming gradient is not only positive but self-reinforcing, governed by a power-law scaling factor $(\delta_g + 1)$.

8.2.3 A comparison with luminosity distance $D_L(\xi)$ of the Λ CDM model

We compared the equation D_{URD} with the standard luminosity distance $D_L(\xi)$ defined in the standard model, which relies on the numerical integration of the Hubble function $E(\xi) = \sqrt{\Omega_m \xi^3 + \Omega_\Lambda}$, where Ω_m is the density parameter of matter (ordinary and dark), and Ω_Λ is the density parameter of the cosmological constant (dark energy):

$$D_L(\xi) = \frac{c\xi}{H_0} \int_1^\xi \frac{d\xi'}{E(\xi')},$$

where c is the speed of light and H_0 is the Hubble constant. This formulation results in a curve that describes the dynamical transition from matter dominance (where the $\Omega_m \xi^3$ term prevails) to dark energy dominance (represented by the constant Ω_Λ). In contrast, the D_{URD} relation reproduces a similar redshift dependence through a direct power-law scaling in ξ , providing an alternative phenomenological parameterization in terms of the parameters α_{U^*} and δ_g .

Comparative analysis of percentage residuals $\Delta\% = 100 (D_{\text{URD}} - D_L)/D_L$ provides critical insight into the scaling evolution of the relation D_{URD} . When examined over an extended range up to $z = 10$ (see Figure 2), the residual curve exhibits a non-linear but highly structured trajectory. Following the initial ‘crossing point’ at $z \approx 1.34$, where the two models achieve a localized numerical equivalence, D_{URD} enters a regime of sustained divergence. By $z = 5$, the positive deviation reaches approximately +25%, escalating to nearly +50% as it approaches $z = 10$.

A fundamental characteristic of this divergence is the downward concavity of the residual function. Whereas the absolute difference between the models continues to grow, the rate at which the percentage deviation increases is strictly decreasing. Mathematically, this is governed by the power-law exponent $\delta_g = 0.251$; since $0 < \delta_g < 1$ (based on Eq. (24) and on in-depth investigations of the parameter in §9.1), the marginal growth of the residual follows a sub-linear trend ($\Delta\% \propto \xi^{\delta_g}$).

This behavior leads to a significant asymptotic flattening of the percentage gradient at high redshifts. As z increases, the curve is laid down, suggesting that the relative discrepancy between D_{URD} and the standard cosmological distance does not undergo an uncontrolled exponential runaway. Instead, the system enters a regime of diminishing marginal deviation.

In conclusion, the D_{URD} relation describes a universe that is intrinsically more ‘expansive’ at high redshifts than predicted by standard Λ CDM integration. However, the downward concavity of the residuals indicates a stable scaling law: the two distance definitions, though diverging in absolute terms, maintain a predictable and slowly-evolving relative proportion.

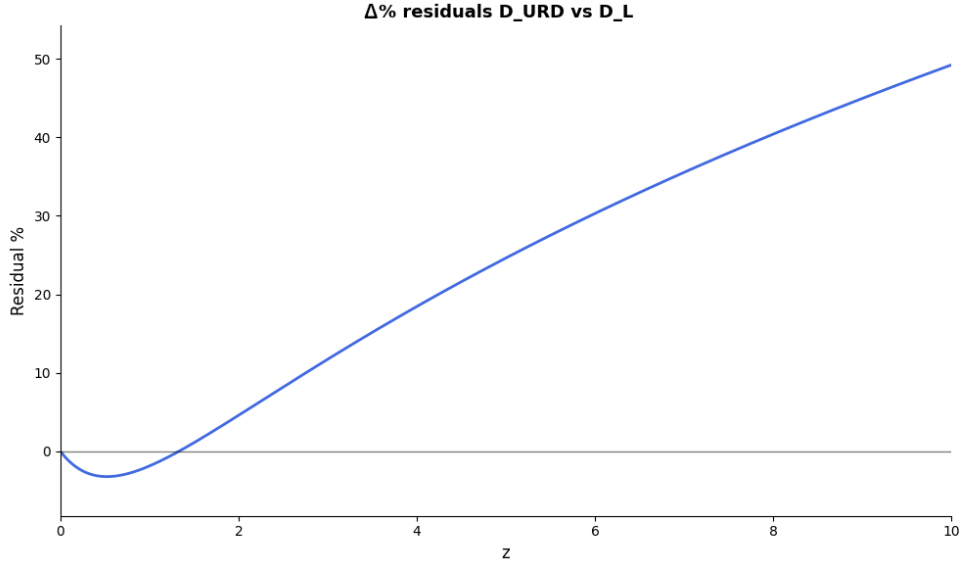


Figure 2: The comparative analysis of percentage residuals $\Delta\% = 100 (D_{\text{URD}} - D_{\text{L}})/D_{\text{L}}$ provides critical insights into the scaling evolution of the D_{URD} relation. Following the initial ‘crossing point’ at $z \approx 1.34$, where the two models of D_{URD} and D_{L} achieve a localized numerical equivalence, D_{URD} enters a regime of sustained divergence.

8.3 Presumed criticalities in the photon energy loss conjecture

The formulated energy loss conjecture is conceptually similar to Fritz Zwicky’s *tired light* hypothesis (Zwicky 1929 [13]). The main criticalities (see, e.g., Wright 2008 [35]) are represented by the interpretation of SNIa light curves (see §8.3.1), by the Tolman test (see Tolman 1930 [36]) on the SB of source galaxies (see §8.3.2), and by the blurring issue (this one excluded in §6).

We have looked at them in detail.

8.3.1 Time dilation

The light curves in the days immediately following the explosion of SNIa have a typical temporal extension, which is observed to depend on the distance or redshift. These periods variable with distance are not considered compatible with the interpretation associated with a *tired light* (see, e.g., Goldhaber et al. 2001 [37]). In the URD framework, it is assumed that the flow of time that is observed depends on the period associated with the frequency of the observed photons. That is, it is considered that the clock defining that event is associated with the oscillation period of the electromagnetic wave of the photon and not that the signal has a ‘physical duration’ independent of the photon (i.e., associated with an ‘absolute clock’). In this way, the principle of causality is preserved without postulating absolute entities. Therefore, if with redshift the wavelength increases, the frequency decreases and the period increases, and the ‘lifetime’ associated with the photon appears longer, compared to a virtual ‘absolute time’⁴. These considerations can be

⁴Regarding the time evolution of photons, the study of Hutchin 2021 [38] is interesting, although different from the URD framework: in the proposed framework, the vacuum field is not fixed but slowly builds up over cosmic time through the cumulative effect of countless excitation and deexcitation processes in stars and other warm bodies, which continuously radiate into it. The strength of this vacuum field is assumed to set the effective value of Planck’s constant h , and thus the local ‘rate of time’: as the vacuum field grows, quantum processes, and therefore clocks, run slightly faster. Light from distant sources was emitted when the vacuum field and time rate were lower than they are today, so photons carry the imprint of a ‘slower’ time at emission and, when measured by faster clocks now, appear redshifted. In this picture, the cosmological redshift arises not from an expanding universe, but from a secular increase in the vacuum field that induces a systematic time-rate difference growing with distance, mimicking a Hubble-like redshift–distance relation.

summarized in the following general relations.

$$\xi = \frac{\lambda_o}{\lambda_e} = \frac{\nu_e}{\nu_o} = \frac{\Delta t_o}{\Delta t_e} = 1 + z \geq 1. \quad (17)$$

It is observed indeed that the photometers, used for observation, use a local clock, but cannot know that the photons arrive with a dilated clock compared to that of the moment in which they were generated; they simply record them as they arrive. Therefore, the observed broadening is naturally reproduced within the URD framework. Rather, it underlines that the fact that they have broadened is fully coherent with the URD model.

In mathematical terms, based on Eq. (17), the observed time delay can be highlighted as:

$$w = \Delta t + [\Delta t_o - \Delta t_e] = \Delta t + [(\xi - 1) \Delta t_e] = (1 + z) t, \quad (18)$$

where w is the time dilation, t_o is the time associated with the moment of observation, and t_e is the time associated with the moment of photon emission, corresponding to the observer's time t .

In a more formal explanation, the relationship between energy loss and temporal stretching can be summarized through the following relativistic framework:

- Energy-frequency coupling: at a fundamental level, the energy of a photon is defined by the Planck-Einstein relation $E = h\nu$; since frequency ν is the inverse of the temporal period between wave crests ($T = 1/\nu$), any reduction in energy E during propagation, regardless of the underlying mechanism, mathematically necessitates a proportional increase in the observed period, based on Eq. (17): $T_o = \xi T_e$.
- Metric invariance and proper time: in GR, the 'tick' of a physical clock is represented by the proper time interval $d\tau$, and according to the spacetime metric, this is expressed as $ds^2 = c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$, where $g_{\mu\nu}$ is the metric tensor that encodes the geometry, gravity, and curvature of spacetime, and dx^μ, dx^ν are infinitesimal shifts in the coordinates (time and three spatial dimensions). For an observer with a fixed spatial coordinate, this simplifies to $d\tau = \sqrt{g_{00}} dt$, where g_{00} represents the temporal component of the metric.
- The identity of redshift-dilation: the ratio of observed to emitted frequency is defined by the metric potentials at the respective points of observation and emission: $\xi = \nu_e/\nu_o = \sqrt{g_{00}(o)/g_{00}(e)}$. Because $\nu = 1/T$, it follows directly that $T_o/T_e = \xi$. This identity confirms that the 'energy loss' (redshift) and the 'slowing of the clock' (time dilation) are not independent processes but two perspectives of the same geometric transformation.

The Eq. (18) is the same theoretical relation developed in Goldhaber et al. 2001 [37], then empirically measured⁵ by them as $w = 0.985 (1 + z)^{(1.045 \pm 0.10)} t$. The most recent study by White et al. 2024 [39] validates that measure with the new more accurate value $w = (1 + z)^{(1.003 \pm 0.005)} t$, which experimentally confirms (and does not refute) the URD model.

A potential critique of the time dilation associated with redshift is that this effect should also be observable in the case of *gravitational redshift*. However, observations confirm exactly the equivalence of redshift and time dilation, for instance, in the case of pulsars in compact binary systems, such as the Hulse–Taylor binary PSR B1913+16 (e.g. Stairs 2003 [40]), or the Double Pulsar PSR J0737–3039A/B (e.g. see Kramer et al. 2006 [41]).

In these systems, the post-Keplerian parameter known as the Einstein delay (γ) quantifies periodic variations in pulse arrival times caused by the combination of transverse Doppler shift and gravitational redshift. As the pulsar reaches periastron, the point of deepest gravitational

⁵The confidence interval (1σ) of the empirically measured exponent goes from 0.945 to 1.145 and therefore contains the theoretical value 1; the coefficient deviates from the theoretical value 1 by a value equal to 1.5 %, compatible with the uncertainty of the experimental measurements.

potential, the energy loss of the emitted photons is observed simultaneously as a stretching of the pulsar's rotational period, by a factor traceable to ξ .

This empirical evidence shows that the redshift is not simply a dissipative effect but a metric-driven scaling of time intervals, ensuring that any energy loss is inherently associated with a proportional temporal dilation.

8.3.2 Tolman test

The original Tolman test on the SB of galaxies (Tolman 1930 [36]) considers the energy flux received per unit of apparent area of the source, $S = F/A$. In the standard cosmological framework, an attenuation factor $\propto (1+z)^{-4}$ is applied, resulting from the combination of photon energy loss $(1+z)^{-1}$, time dilation $(1+z)^{-1}$, and the apparent increase in the observed area $(1+z)^{-2}$ due to cosmic expansion. In the URD framework, the factors that reduce the flux are only the first two, and the standard Mattig's equation (Mattig 1958 [42]) that connects the redshift to luminosity distance D_L , which in this case equals the proper distance D_{URD} , is not applicable (the deceleration parameter becomes indefinite). Therefore, we utilize the distance modulus (see Eq. (19)–(22)) and the proper URD distance D_{URD} to derive a verifiable geometric scaling for galactic SB.

The URD framework posited that the observed distance-redshift relationship is a consequence of a static geometric progression, therefore, we consider the SB, which provides a more direct influence of the underlying geometry. In a non-expanding universe where the reciprocity relation is governed solely by photon energy loss, the luminosity distance (D_L) and the angular diameter distance (D_A) are related by a factor of $(1+z)$. Consequently, SB is expected to decrease only with the second power of $(1+z)$, leading to the following theoretical derivation.

$$\text{SB}_{\text{URD}}(z) = \text{SB}(0) + 2.5 \log(1+z)^2 = \text{SB}(0) + 5 \log(1+z) .$$

This equation translates the URD radial geometry into a SB prediction, allowing a quantitative comparison against observational data from different astronomical tracers and eras.

The Tolman test is conceptually straightforward but historically plagued by two classes of issues: galactic evolution and technical biases. In standard cosmology, the apparent failure of the $(1+z)^{-4}$ law at high redshift is typically addressed by assuming significant luminosity evolution, where galaxies in the early universe are hypothesized to be intrinsically brighter than their modern counterparts. Furthermore, technical factors such as the K-Correction⁶ and surface dimming⁷ due to instrumental limits can skew the results. In the study by Lubin and Sandage 2001 [43], considerable effort was spent to estimate these parameters, but the potential for associated biases remains high when interpreted within different general models.

As a first observation, we note that the distance modulus DM_{URD} produces values very close to the standard luminosity distance (D_L) in a Λ CDM cosmology, with a mean theoretical difference $\langle \Delta\mu_{\text{gap}} \rangle = \langle \Delta\mu_{\text{std}} - \Delta\mu_{\text{URD}} \rangle \sim -0.44\%$.

From a theoretical point of view, at a redshift of $z = 0.92$ (corresponding to the cluster 1604+4321 in Lubin & Sandage 2001 [43]), we can calculate the 'gap' $\Delta\mu_{\text{gap}} = 1.42$ mag between the attenuation factor for SB $\Delta\mu_{\text{std}}$ with the standard $1/(1+z)^4$ and the $\Delta\mu_{\text{URD}}$ with $1/(1+z)^2$:

$$\begin{aligned} \text{Standard prediction: } \Delta\mu_{\text{std}} &\approx 10 \lg(1.92) \approx 2.84 \text{ [mag arcsec}^{-2}\text{]} \\ \text{URD prediction: } \Delta\mu_{\text{URD}} &\approx 5 \lg(1.92) \approx 1.42 \text{ [mag arcsec}^{-2}\text{]} \quad . \end{aligned}$$

⁶Because, in the standard model, expansion shifts light toward the red, telescopes observe a different part of the electromagnetic spectrum than what was originally emitted; hence precise knowledge of the source's spectrum is required to compare 'rest-frame' brightness across different distances.

⁷As galaxies recede, their faint outer edges drop below the sky's background noise; this leads to an underestimation of their true size or total luminosity, which can skew the calculated SB.

To reconcile the URD model with the observational data of Lubin & Sandage 2001 [43] (who claim consistency with the $(1+z)^{-4}$ law), we analyze the light distribution of elliptical galaxies (with Sérsic 1963 [44] profile⁸ with index $n = 4$).

The URD model posits that the ‘surplus’ of 1.42 magnitudes of luminosity predicted by the $(1+z)^{-2}$ law are not missing but *undetected*. In a de Vaucouleurs profile (de Vaucouleurs 1948 [45]), approximately 70 % of the total luminosity resides in the extended, low-surface-brightness ‘wings’ of the galaxy.

At high redshifts, the Sky Background noise (typically $\mu_{\text{sky}} \approx 22.5 \text{ mag arcsec}^{-2}$ for HST data) acts as a physical threshold.

Hypothesized mechanism: in the URD model, galaxies at $z \approx 0.9$ are physically more compact but intrinsically brighter than the standard model assumes (see also §8.2). However, the $(1+z)^{-2}$ dimming, combined with the URD geometry, places the outer 70% of the galaxy’s light below the detectable sky noise. The loss of $\sim 70\%$ of the total flux due to background noise truncation results in an observational dimming of ~ 1.45 magnitudes. This value approximately offsets the URD ‘surplus’, causing the measured magnitude to mimic the standard $(1+z)^{-4}$ curve. This ‘geometric shrinkage’ creates an apparent consistency that appears to support the expanding universe hypothesis, whereas it can be interpreted as with a non-expanding geometry.

A note on the mathematical failure of the Petrosian radius as an invariant

Lubin & Sandage 2001 [43] rely on the Petrosian radius (θ_P , see Petrosian 1976 [46]) under the assumption that it provides a metric-independent measure of a galaxy’s extent.

Theoretically, for a Sérsic profile with $n = 4$, the Petrosian ratio $\eta(r)$, defined as the ratio of the local SB to the average brightness within that radius, should reach the standard value of 0.2 at approximately $2.15 R_e$. However, this mathematical invariance collapses in the high-redshift, high-noise regime ($z \approx 0.9$):

- In real-world observations, the measured ratio becomes:

$$\eta_{\text{obs}}(r) = \frac{I(r) + B_{\text{sky}}}{\langle I(< r) \rangle + B_{\text{sky}}} ,$$

where B_{sky} represents the sky background. As the SB $I(< r)$ of the galaxy’s outer wings approaches the level of the background noise, the ratio $\eta(r)$ is ‘padded’ by the constant B_{sky} term. This prevents the ratio from smoothly declining toward the 0.2 threshold.

- Because in the URD model the galaxy has a larger apparent angular extent in the sky, since the $(1+z)^{-2}$ factor is absent, its low-SB wings spread over more pixels and fall below the sky background sooner. Consequently, detection algorithms identify the limit $\eta = 0.2$ much closer to the high-intensity galactic core, often at $r \approx 0.7 \div 1.0 R_e$ instead of the theoretical limit $2.15 R_e$.
- This ‘geometric shrinkage’ creates a dramatic bias: the observer measures a θ_P/R_{50} ratio that collapses toward 1.0, effectively discarding up to 70–80 % of the total flux residing in the truncated wings. By measuring only the ‘bright core’ whereas assuming that it represents the whole galaxy, the final calculation of SB is artificially skewed, providing an interpretation that mimics the standard $(1+z)^{-4}$ expansion law.

⁸From the Sérsic profile for the SB, defined as $\ln I(r) = \ln I_0 - k r^{1/n}$, where r is the galactocentric radius and I_0 is the central surface brightness. Here, k is a scale factor related to the effective radius (the radius enclosing half of the galaxy’s total luminosity), whereas n is the Sérsic index. This index acts as a ‘shape’ parameter that determines the profile’s curvature: a high value of n implies a steep central peak with extended wings, whereas a low value signifies a flatter core. The radial term $r^{1/n}$ dictates the specific brightness decay rate. In particular, an index of $n = 4$ recovers the de Vaucouleurs profile, $I(r) \propto \exp(-k r^{1/4})$.

8.3.3 Analysis of the Tolman test observational data

Table 2 summarizes the results for a sample spanning from the local universe to extreme redshifts ($z \sim 13$). As observed in HST data (e.g., Lubin and Sandage 2001 [43]) and, more recently, in JWST data (e.g., Lovyagin et al. 2022 [6]), the original cluster data are compatible with the phenomenological comparison performed here. The results of elliptical clusters confirm that the URD model, when combined with instrumental truncation, provides an excellent fit with differences below 1.1 %. In contrast, the analysis of disk galaxies (Lerner et al. 2014 [7]) and recent JWST data in Lovyagin et al. 2022 [6] reveals that the rest-frame ultraviolet SB remains nearly constant, varying only by 0.3 mag from $z = 0$ to $z = 5$, whereas galaxies at $z > 10$ appear significantly more luminous than standard optics predict.

As independently confirmed by Lovyagin et al. 2022 [6], if the URD-consistent $(1+z)^{-2}$ scaling factor is applied to luminosity-based mass estimates, the physical characteristics and morphologies of early galaxies become statistically identical to those in the local universe. That is, galaxies at high redshift appear significantly smaller and more luminous than predicted by the Λ CDM model. Specifically, the standard model requires galaxies at $z > 10$ to be physically much smaller but proportionally more massive (up to 1/10th the size of local counterparts) to satisfy expansion optics: a requirement that lacks a physical basis in current galaxy formation theories. Lovyagin et al. 2022 [6] results remove this requirement for ‘impossible’ tiny but massive galaxies, and is consistent with the observed brightness anomaly as evidence of a non-expanding geometry.

Table 2: Tolman test analysis.

Source / Sample	Redshift z	SB Median [mag arcsec $^{-2}$]	SB _{URD} [mag arcsec $^{-2}$]	Δ SB %	Note / Instrument
Galex ^(a)	0.03	24.39 ± 0.03	24.39	0	Disk gal. / UV Tel. local calib.
Cluster 1324 ^(b)	0.76	~ 22.6	22.37	−1.02	Cluster HST (R-band)
HUDF Bin 1 ^(c)	0.82	24.28 ± 0.08	25.63	5.56	HST Ultra Deep Field
Cluster 1604 ^(b)	0.90	~ 22.6	22.54	−0.27	Cluster HST (R-band)
Cluster 1604 ^(b)	0.92	~ 22.56	22.56	0	Cluster HST (R-band)
HUDF Bin 2 ^(c)	1.83	24.18 ± 0.10	26.58	9.93	HST Ultra Deep Field
HUDF Bin 3 ^(c)	3.52	24.08 ± 0.11	27.60	14.62	HST Ultra Deep Field
HUDF Bin 4 ^(c)	4.83	24.08 ± 0.10	28.15	16.90	HST Limit (UV-flat)
JWST Deep Field ^(d)	$\sim 10\text{--}13$	~ 24.39	30.06	23.25	$\times 14$ brighter than Λ CDM

^(a) Lerner et al. 2014 [7], Table 1; ^(b) Lubin & Sandage 2001 [43], Table 1; ^(c) Lerner et al. 2014 [7], Table 2; ^(d) Lovyagin et al. 2022 [6], Table 1.

8.3.4 Conclusions on Tolman test

The URD model shows twofold consistency with cosmological observations. The distometric analysis aligns with the standard supernova distance scale, whereas the SB test suggests the consistency of a non-expanding geometric progression. By providing a unified physical explanation for the observed magnitude gap as an effect of profile truncation and instrumental limits, the URD framework may account for distant tracers into a single, expansion-free framework.

In light of the observations of JWST, the analyzes of Lovyagin et al. 2022 [6] and the URD framework, this area of research certainly deserves further investigation in a dedicated study in the future (e.g., utilizing the high-quality SB data Beltrán et al. 2024 [10]).

9 URD parameters calibration with Pantheon+ dataset

In this Section, we calibrate the parameters α_{U^*} and δ_g of the Eq. (15) and cross validate them with a detailed statistical analysis.

9.1 Calibration of the URD equation with Pantheon+ dataset

The observational data extracted from the online archives⁹ on which The Pantheon + Analysis is based by Brout et al. 2022 [47] were used to calibrate and test Eq. (15), and the URD model. They are 1,701 records of 1,543 supernovae Ia type in a wide redshift range ($0.0012 \leq z \leq 2.26$).¹⁰

The data extracted for each source are the redshift (ZCMR) calculated with respect to the rest frame of cosmic microwave radiation and the distance modulus (DM), with their measurement errors (see Table 3 and `_file1.csv`, available for download, along with the other project files, on the dedicated page of our website: <https://andreabrussi.it/data-download>).

Table 3: Contents of `_file1.csv`: observational data and calculated fields for Pantheon+ dataset (header and first 5 records).

NUM	CID	IDSURV	ZCMR	ZCMR_ERR	DM	DM_ERR	D_URD	D_DM
1	2011fe	51	0.00122	0.00002	28.9987	1.51645	5.0521	6.3058
2	2011fe	56	0.00122	0.00002	29.0559	1.51747	5.0521	6.4741
3	2012cg	51	0.00256	0.00002	30.7233	0.782372	10.6099	13.9528
4	2012cg	56	0.00256	0.00002	30.7449	0.799068	10.6099	14.0922
18	2013dy	51	0.00293	0.00012	31.499	0.514259	12.1462	19.9434
...								

NUM*: original record number; **CID***: object name; **IDSURV***: survey reference; **ZCMR***: redshift in CMR frame; **ZCMR_ERR***: redshift error; **DM***: distance module; **DM_ERR***: distance module error; **D_URD**: distance calculated with Eq. (15) and Eq.(24); **D_DM**: distance calculated with Eq. (23). * means Pantheon+ original data.

As a reminder, let us recall that the equation for the distance modulus $DM = (m - M)$, which represents the difference between apparent magnitude (m) and absolute magnitude (M), considering the distance in Mpc, is:

$$DM = 5 \lg(D_L) + 25 , \quad (19)$$

$$DM_{URD} = 5 \lg(D_{URD}) + 25 . \quad (20)$$

For completeness, the relation with corrective parameters (SALT2 model) is also added:

$$DM = m_B - M_B + \alpha' \chi_1 - \beta c , \quad (21)$$

where m_B is the apparent magnitude at maximum in the rest frame B band, M_B is the absolute magnitude of a ‘standard’ SnIa (with $\chi_1 = 0$ and $c = 0$), α' is the coefficient that links to their luminosity the stretch of the light curves χ_1 of SnIa (Phillips effect, Phillips 1993 [50]) and β coefficient accounts for the color correction (because ‘bluer is brighter,’ redder SnIa, which have a positive c value, appear dimmer, due to a combination of intrinsic Sn properties and interstellar extinction).

⁹ <https://github.com/PantheonPlusSH0ES/DataRelease>

¹⁰ A note on redshift in Pantheon+ database: The Pantheon+ compilation achieves unprecedented redshift accuracy by prioritizing host-galaxy spectroscopy over supernova-derived values, effectively decoupling the distance indicator from the cosmic expansion measurement of the cosmological standard model. As detailed by Carr et al. 2022 [48], this process involved a systematic cross-verification with external catalogs, including radio surveys of the 21 cm line and distances derived from the Tully–Fisher relation (see Tully & Fisher 1977 [49]), ensuring that host redshifts are consistent across different physical tracers. To mitigate the impact of local motion, redshifts were further corrected using 2M++ density field maps and assigned to group-center values, reducing the bias from nonlinear peculiar velocities. These refined, galaxy-based measurements, independent of the Sn light-curve physics, were ultimately integrated into the Brout et al. [47] database, providing the robust kinematic foundation necessary for the high-precision determination of H_0 Hubble constant, all this in the framework of the standard cosmological model.

Based on Eq. (15), equation Eq. (20) reads:

$$DM_{\text{URD}} = 5 \lg \left[\frac{\xi (\xi^{\delta_g} - 1)}{\alpha_{U^*} \delta_g} \right] + 25. \quad (22)$$

With these equations, the distance from the distance modulus was calculated (the values D_{DM} reported in `_file1.csv`):

$$D_{\text{DM}} = 10^{(DM-25)/5} \text{ [Mpc]}. \quad (23)$$

Management of duplicate data

To ensure the statistical independence required for valid results, we merged duplicate records for the same supernova (coming from different surveys) using an inverse-variance weighted average.¹¹ For each unique supernova, we calculate the average by weighting each observation x by the inverse of its squared error ($1/\sigma^2$). The equation for the weighted average is: $\langle x \rangle = \sum (x_i/\sigma_i^2) / \sum (1/\sigma_i^2)$, where: x_i is the individual measurement; σ_i is the error (uncertainty) of that measurement; $\sum$ represents the sum of all terms.

9.1.1 Optimization algorithm

Our optimization algorithm was specifically designed to fit the parameters α_{U^*} and δ_g in the Eq. (15): rather than the standard least-squares fitting on distances, the primary goal of this optimization is to ‘flatten the residual trend’. Specifically, it minimizes the slope of the percentage deviation between the theoretical distance (D_{URD}) and the empirical distance (D_{DM}) derived from the observed distance modulus.

The objective function to minimize is the absolute slope of the linear regression (see Theil 1950 [51]) $\Delta\% = 100 (D_{\text{URD}} - D_{\text{DM}})/D_{\text{DM}}$ vs redshift z . The procedure involves the following steps:

- Data aggregation: the 1,543 unique SnIa records from the Pantheon+ database are grouped by their unique redshift z , as detailed in the above “Management of duplicate data” Section.
- Systematic Grid Search: A 1D grid search is performed over the non-linear shape parameter δ_g .
- Analytical least squares: For every discrete step of δ_g tested on the grid, the linear scaling parameter α_{U^*} is solved analytically. Taking into account $1/\alpha_{U^*}$ from the URD equation, the problem simplifies to $y = mx$, where $m = 1/\alpha_{U^*}$. A simple least squares fit through the origin determines the analytically determined best-fit value α_{U^*} for that specific δ_g and the given dataset.
- Trend minimization: The combination of δ_g and its analytically derived α_{U^*} that produces a slope of the residual percentage deviation naturally closest to zero is selected as the optimal pair.

9.1.2 Parameter measurements and error estimation

To properly compute the statistical errors (since standard covariance matrices are not easily derived from a trend-minimization objective), we have selected a search grid¹² of ± 0.25 , to

¹¹This methodology does not have a precise paternity but can be traced back to the studies of Simone Laplace and Carl Gauss, who developed the method of least squares at the end of the 18th century.

¹²We performed a sensitivity test on the grid used for the bootstrap analysis, using two other different grids:

A grid ± 0.50 : the comparison between the original baseline and the expanded reviewer range yields near-identical

find the true unrestricted minimum and performed a bootstrap analysis (Efron 1979 [52]) with 500 random resamplings of the dataset with replacement.

The unrestricted optimal values and their standard errors 1σ (derived from the variance in the bootstrap distribution) were found to be the following:

$$\left[\begin{array}{l} \alpha_{U^*} = (2.4167 \pm 0.0004) \times 10^{-4} [\text{Mpc}^{-1}] \\ \delta_g = 0.251 \pm 0.020 \end{array} \right], \quad (24)$$

The appropriate distances D_{URD} reported in `_file1.csv` were determined with these values and Eq. (15).

In Figure 3 we have reported the behavior of the Eq. (15) with respect to the distances D_{DM} based on the distance modulus Eq. (23), well interpolated without additive correction for the presumed acceleration of the presumed expansion.

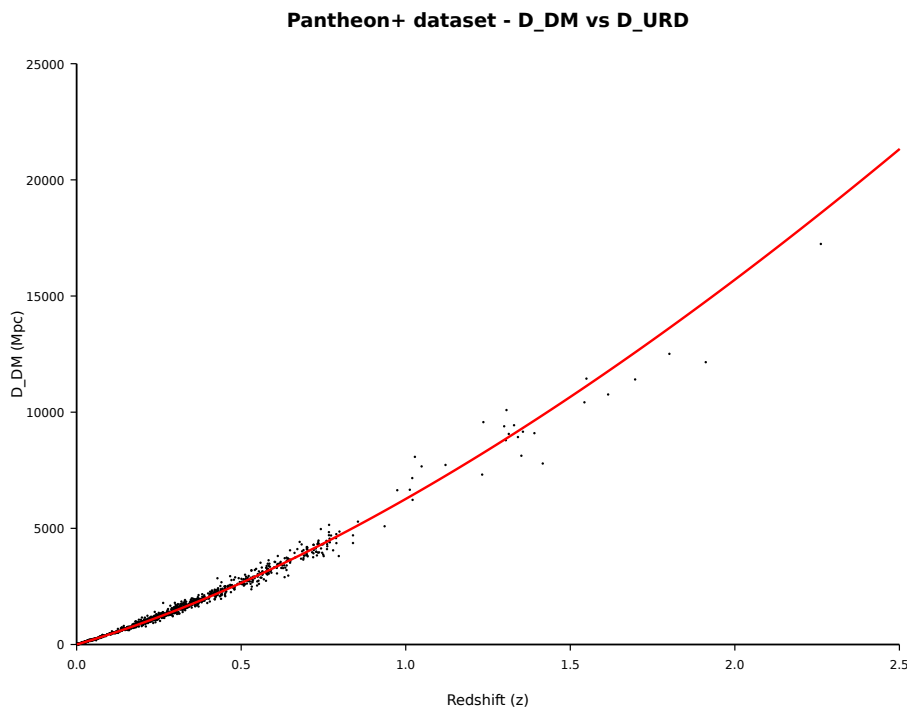


Figure 3: In the graph we have reported the trend D_{URD} of the Eq. (15) with respect to the distances D_{DM} derived from the Pantheon+ database by means of the distance modulus Eq. (23); the curve obtained from the redshift interpolates well the distances that are not based on it. See also Figure 4.

In Figure 4 we have reported the same data as in Figure 3 but only where $z > 0.9$, to visualize error bars; for lower values, there would be too many error bars to visualize.

Values exceeding the observational uncertainty of $\pm 14.54\%$ (2σ) are found mostly within the range Eq. (26), corresponding to distances up to $D_{\text{URD}} \approx 42$ [Mpc]; for a detailed analysis of this aspect, see §9.4.

results, with the optimal $\delta_g \sim 0.25$; this consistency proves that the URD model converges to a single, well-defined physical minimum across different scales, regardless of the initial search boundaries.

A grid ± 0.125 : when the model is restricted to the local geodesic value of 0.125, the optimization is forced against the boundary limit (therefore, the algorithm had to stop at the imposed limit, without being able to reach the ideal value of $\delta_g = 0.25$), resulting in a $\chi^2_{\text{red}} = 0.532$. The fact that the increase in the fit residual is only approximately 2.3% (relative to the global minimum of $\chi^2_{\text{red}} = 0.520$, which is the χ^2_{red} value calculated on the distance, equivalent to the previously calculated value $\chi^2_{\text{red}} = 0.463$, based on the distance moduli) provides a good defense for the URD framework; it shows that the model maintains good statistical consistency even when constrained to local parameters, proving its robustness against secondary geodesic variations.

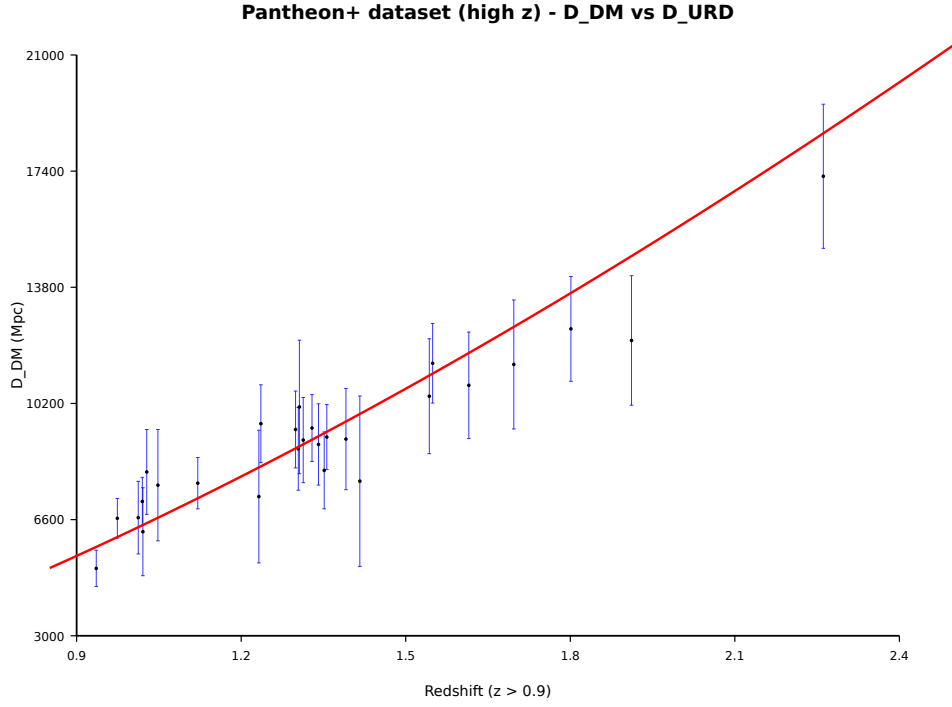


Figure 4: In the graph we show the same data of Figure 3 (Pantheon+ database analysis) but only where $z > 0.9$, with error bars; for lower values there would be too many error bars to visualize.

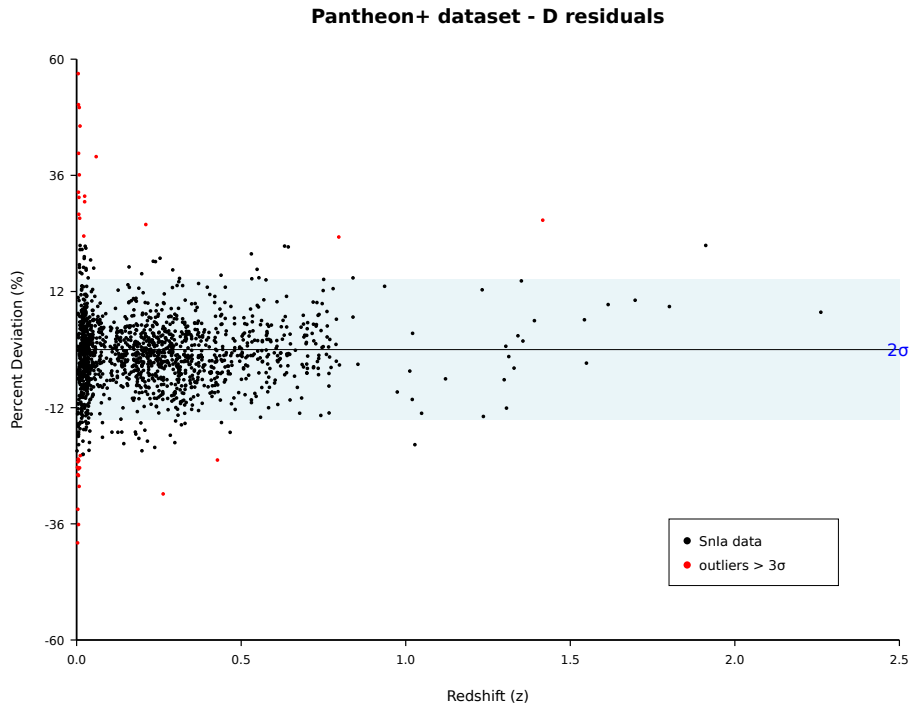


Figure 5: In the graph, the distribution of fractional distance residuals between the distances calculated with distance modulus and the ones calculated with URD model (Pantheon+ database analysis). The successful calibration of the Eq. (15) is evidenced by the centering of points around the zero-deviation line across the entire dataset. The shaded blue band highlights the (2σ) observational uncertainty $\pm 14.54\%$, and the red dots the outliers.

In Figure 5 the distribution of fractional distance residuals between the distances calculated with the distance modulus and those calculated with the URD model is reported. The successful

calibration of the Eq. (15) is evidenced by the centering of points around the zero-deviation line across the entire dataset.

For a further comparison between the distances, in Figure 6 we plotted a log-log correlation between the standard distance modulus measurements D_{DM} and the distances derived from the refined URD model D_{URD} .

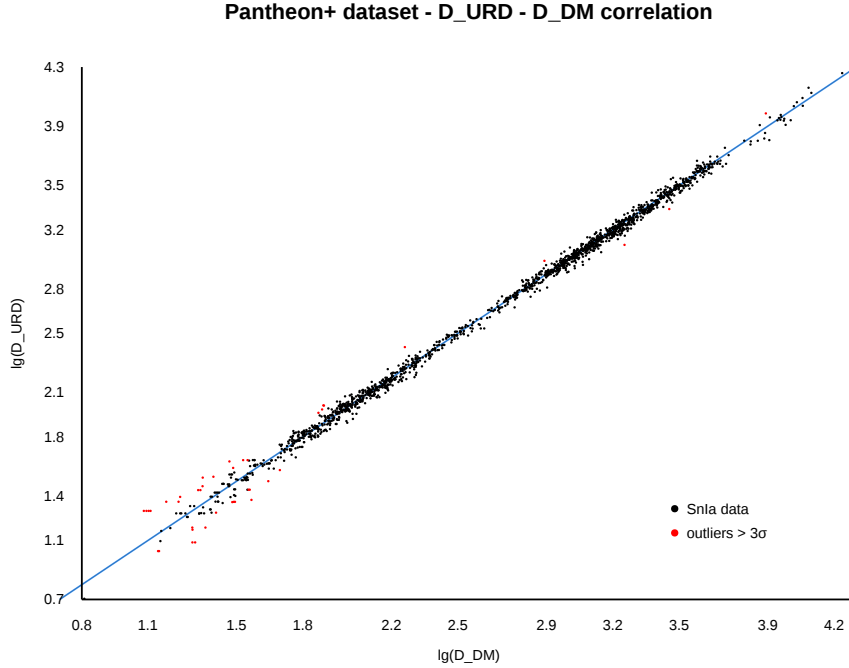


Figure 6: In the plot a log-log correlation between standard distance modulus measurements D_{DM} and refined URD model distances D_{URD} , red points are outliers exceeding 3σ (Pantheon+ database analysis).

9.2 A cross-validation study on the URD equation applied to the Pantheon+ dataset

We calibrated the empirical parameters of the URD model exclusively on a subset of low-redshift objects, of the Pantheon+ database, to test its ability to accurately predict the distances of high-redshift supernovae, thus confirming the structural consistency of the URD distance Eq. (15).

To ensure data consistency, records referring to the same supernova were aggregated using an inverse-variance weighting method (see §9.1). The dataset was divided into two distinct sets:¹³

- Training set: comprises the first 500 unique supernovae; this subset covers the low-redshift range from $z = 0.0103$ to $z = 0.0825$.
- Test set: comprises the remaining 970 Sn, extending from $z = 0.0825$ up to $z = 2.2613$.

¹³For the rigorous estimation of the model's statistical variance (the 2σ tolerance band), the subset of supernovae at very low redshift ($z \leq 0.01$, Eq. (26)) was analytically excluded (that means 74 Sn less). This strict minimum redshift cut-off is a standard procedure in modern observational cosmology: at such short cosmic distances, the underlying radial cosmological velocity is severely contaminated by the peculiar velocities of the host galaxies, driven by local gravitational flows. Including these chaotic local kinematic data points would artificially inflate the standard deviation, severely mischaracterizing the true precision of the cosmological fit at deeper scales. This exact procedural cut-off ($z_{\text{min}} \sim 0.01$ to 0.023) is actively enforced in the calibration of the standard Λ CDM framework and the Hubble constant measurements. As references: Riess et al. 2016 [53] applied a $z > 0.023$ cut-off to minimize peculiar velocity errors; Brout et al. 2022 [47] discussed the strict low-redshift cut-offs and peculiar velocity corrections required for robust cosmological fitting.

The empirical parameters $\alpha_{U^*}^{500}$ and δ_g^{500} were optimized by minimizing the unweighted rmse in the training set.

The optimization process on the low redshift calibration set yielded the following values:¹⁴

$$\begin{aligned}\alpha_{U^*}^{500} &= (2.39 \pm 0.06) \times 10^{-4} \text{ [Mpc}^{-1}\text{]} \\ \delta_g^{500} &= 0.135 \pm 0.038 .\end{aligned}$$

Test on training set: $\chi^2 = 305.77$; $\chi_{\text{red}}^2 = 0.614$; BIC = 318.20.

Although the absolute value of χ^2 reflects the sample size ($N = 500$), the fact that the reduced χ^2 (χ_{red}^2) is less than unity suggests a conservative estimate of the uncertainties in the original data. Although such a condition could sometimes point to overfitting, the Bayesian Information Criterion (BIC, Schwarz 1978 [54]) value of 318.20 confirmed the statistical parsimony and consistency of our two-parameter model.

The uncertainties are calculated using the $\Delta\chi^2 = 1$ criterion, which corresponds to a 1σ confidence interval. These parameters represent the ‘local’ geometry of the URD model where luminosity-related biases are minimal (see the considerations in §8.2).

The model, calibrated on the low-redshift subset, was evaluated against the remaining high-redshift sample (970 Sn). Despite the limited calibration range, the predictive power is remarkably high, as shown by the analysis of the residuals from the comparison with the distances calculated using the distance modulus:

- Mean percentage of residual: -1.46%
- Standard deviation of residuals: 6.56%
- $\chi^2 = 408.37$; $\chi_{\text{red}}^2 = 0.421$; BIC = 422.13
- rmse = 205.06 Mpc .

9.2.1 Comparison with measurement uncertainties

For the high-redshift test set, the average measurement uncertainty in distance is approximately 222.43 Mpc. The model’s predictive error (rmse = 205.06 Mpc) is lower than the average instrumental error bar of the dataset itself.

Since the reduced χ^2 for the prediction (0.421) is significantly below unity, it confirms that the differences between the predictions of the URD model and the observed distances are, on average, smaller than the stated experimental uncertainties. Therefore, we may say that the URD Eq. (15) captures the global cosmological trend using only local geometric information.

The stability of the parameter α_{U^*} in all tested configurations (varying about 1% from the global value in Eq. (24)) further corroborates its role as a fundamental scale constant for the model.

9.2.2 Correlation Analysis (α_{U^*} vs δ_g)

Based on the full sample of 1543 unique supernovae, the correlation coefficient is $\rho_{\text{URD}} = +0.7554$. There is a good positive correlation between α_{U^*} (governing the global distance scale) and δ_g (the power-law correction): a variation in α_{U^*} is statistically compensated for by δ_g to maintain a consistent fit across all redshifts.

¹⁴Note that the error in delta is large because sensitivity to this parameter is lower at low redshifts, confirming its role as a ‘correction’ for large distances.

9.2.3 On the variation of parameter δ_g

The variability of the geodesic parameter δ_g between different redshift bins provides a clear physical insight into the model's behavior. In the local universe ($z < 0.08$), the optimal value is approximately 0.125, suggesting that the geodesic correction is less necessary for sources within our immediate neighborhood. However, as light travels across longer cosmic paths, the parameter increases and eventually stabilizes near the global mean value of 0.251 (peaking between 0.24 and 0.29 in the intermediate range $0.08 < z < 0.5$). This trend should not be viewed as a need for equation revision, but rather as an empirical convergence: as the photon's path increases, the geodesic correction consistently approaches the stable global geometry of the URD model. The stability of the parameter α_{U^*} (within 1 %) throughout this transition confirms that the fundamental scale of the model remains universal, whereas the evolution of δ_g simply reflects the transition from the local flow to the large-scale structure of the universe.

9.2.4 Conclusions on cross-validation

The cross-validation experiment using 500 Sn as a training sample confirms the high predictive power of the power law Eq. (15). By excluding the $z \leq 0.01$ region contaminated by peculiar velocities (see footnote 13), the model shows excellent stability, extrapolating from the local flow to the high-redshift universe with precision constrained only by the intrinsic noise of the data set.

9.3 Statistical error propagation analyses in Pantheon+ dataset

We performed a quantitative check to verify whether the residual percentage deviations of the model follow a Gaussian distribution, as expected for purely random measurement errors. The calculations confirm that the distribution of the residuals is highly concentrated around the mean (leptokurtic), showing a significant stability of the global fit; see Figure 7 comparing the empirical distribution with a normalized Gaussian curve (where the area under the curve matches the total number of samples times the width of the bin).

Based on the parameters in Eq. (24), the statistical metrics for the percentage deviations, defined as $(D_{\text{URD}} - D_{\text{DM}})/D_{\text{DM}}$, across the entire Pantheon+ sample are:

- average residual = -0.69%
- standard deviation = 8.42%
- skewness = 1.00 (positive skewness)
- kurtosis 7.32 (significant central density)
- Jarque–Bera test: $\text{JB} = 3703.74$ ($p\text{-value} \approx 0$).

The distribution shows a density in the central sigma significantly higher than a standard normal distribution, with 78.7 % of the data within 1σ . This indicates that there is no significant evidence of systematic bias across the global scale, as the residuals are centered on a near-zero mean. The non-Gaussianity (high kurtosis) is not an intrinsic flaw of the nonlinear URD model but is inherited directly from the standard candle assumption applied to the heterogeneous Pantheon+ dataset.

The inherent non-Gaussian nature of the underlying data affects the empirical distances in critical ways:

- Non-linear exponential bias: the exponential transformation creates an intrinsic mathematical bias; because the exponential is a convex function, positive variations in the distance modulus result in a massive proportional increase in the calculated distance D_{DM} in Megaparsecs.

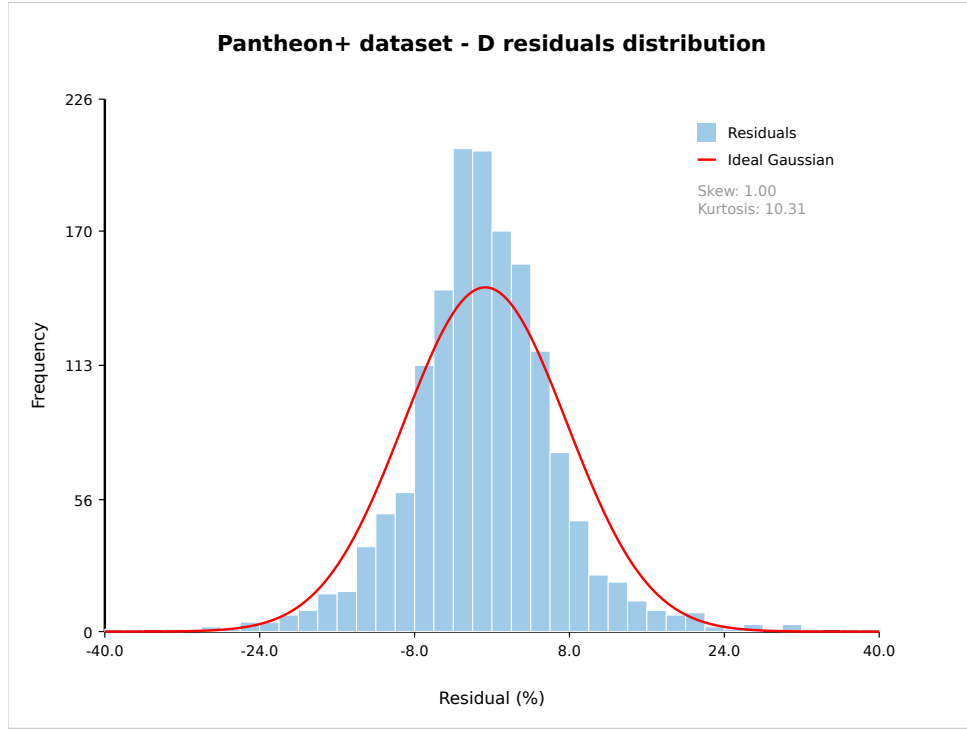


Figure 7: The plot, based on the Pantheon+ database, compares the empirical distribution of residuals, between the distances calculated with distance modulus and the distances calculated with URD model, with an ideal Gaussian curve. The presence of ‘fat tails’ is clearly visible, i.e., a higher than expected occurrence of outliers.

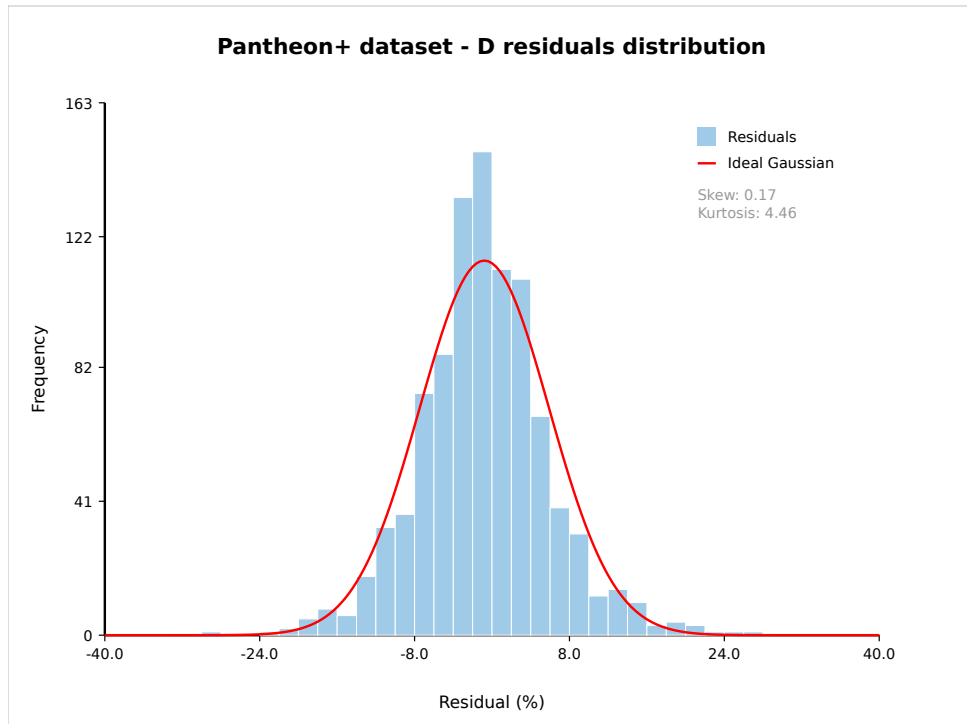


Figure 8: The plot, based on the Pantheon+ database, compares the empirical distribution of residuals with $z > 0.1$, between the distances calculated with distance modulus and the distances calculated with URD model, with an ideal Gaussian curve. In this case the fitting is much better than in Figure 7.

- The statistical impact of heavy tails: the dataset contains far more extreme central concentrations and outliers than a normal distribution allows; consequently, the global fit must

manage a dispersion that is dominated by the noise of individual supernova measurements.

9.3.1 The High-Redshift Gaussianity Recovery ($z > 0.1$)

To isolate the source of this noise, we filter the data set to exclude local sources ($z < 0.1$), minimizing the impact of peculiar velocities and local bulk flows (see Figure 8).

In this deep cosmological regime ($N = 960$), the statistical clarity of the URD model increases significantly:

- average residual = -0.79%
- standard deviation = 6.67%
- The skewness drops to 0.17 (almost perfectly symmetric)
- kurtosis drops significantly to 1.46 (drastically reduced tails)
- Jarque–Bera test: $JB = 90.58$ (massive reduction compared to global).

In the deep cosmological regime ($z > 0.1$), the empirical data recover a much cleaner, symmetric Gaussian distribution. The reduction of the sigma to 6.67% is a fundamental qualitative leap: this value is remarkably close to the intrinsic instrumental noise floor of the Pantheon+ dataset, suggesting that the model correctly follows the curvature of the observational data without systematic bending.

9.3.2 Conclusions on the URD parameters

When we fit the strictly geometric URD power law Eq. (15), optimization shows profound structural stability.

The geodesic correction parameter ($\delta_g = 0.251$) is the direct mathematical manifestation of the model capturing the non-linear path correction at high redshifts. The residuals define a nearly symmetric distribution (skewness 0.17) in the cosmological regime, indicating that the URD Eq. (15) captures the radial-distance trend in a stable and internally consistent way, without evidence of a significant systematic drift between observed and predicted distances. The reduction of the average error to less than 7% further supports the statistical adequacy of the URD distance model over the range considered.

9.3.3 A global χ^2 analysis

This analysis optimizes the model's parameters across all available observations to assess its overall statistical consistency. The minimized objective function is the statistic $\chi^2 = \sum [(D_{\text{URD}} - D_{\text{DM}}) / \sigma_{\text{DM}}]^2$. The uncertainties of the parameters are rigorously determined using the criterion $\Delta\chi^2 = 1$, which defines the 1σ confidence interval.

The global optimization yielded the following parameters, in addition to the values of Eq. (24) (with 1541 degrees of freedom, 1543 unique Sn minus 2 parameters):

- $\chi^2 = 714.27$; $\chi^2_{\text{red}} = 0.463$; $\text{BIC} = 728.95$
- $\text{rmse} = 144.55 \text{ Mpc}$.

The reduced value χ^2 , which being significantly below unity, indicates that the URD model interpolates the whole Pantheon+ data with a precision that exceeds the stated experimental uncertainties (as does the ΛCDM model, with a $\chi^2_{\text{red}} < 1$, see Keeley et al. 2024 [55]). This confirms that the equation captures the cosmological trend with good stability (see also the note 13), reducing the residual dispersion below the instrumental noise level. This confirms that Eq. (15) is not only a good fit but is highly stable against the intrinsic noise of supernova observations.

9.4 Analysis of measurement dispersion at low redshift in Pantheon+ database

Taking into account the observational data extracted from the Pantheon + database, we can see that the dispersion of the calculated redshift is particularly evident in the range $z \leq 0.01$ or $\log z \leq -2$, see Figure 9. In this range, the highlighted outliers are the data that exceed 3σ . Various studies have been published to suggest improvements in the processing methodologies of the Pantheon+ database in the case of very low redshift and to reduce data dispersion; in particular:

- Peterson et al. 2022 [56] focus on the two dominant ‘motions’ contributing to nearby peculiar velocities: large-scale, coherent-flow motions and small-scale motions due to gravitationally associated galaxies deemed to be in a galaxy group; they find that the optimal flow corrections derived from various local density maps significantly reduce Hubble residuals or the dispersion of distance measurements;
- Wojtak & Hjorth 2022 [57] reanalyze the slopes of the color correction in the calibration sample for SNIa standardization, finding that the local measurement of the Hubble constant becomes dependent on the choice of SNIa reference color and indirectly changes the estimate of the absolute magnitude value, which can directly lead to a larger dispersion in inferred distances for the sources;
- Perivolaropoulos & Skara 2023 [58] introduce the systematic volumetric redshift scatter bias and a mild signal for a change of intrinsic SNIa luminosity at about 20 Mpc; they use a likelihood model for the absolute magnitude parameter, which dramatically changes the quality of fit to the Pantheon+ sample.

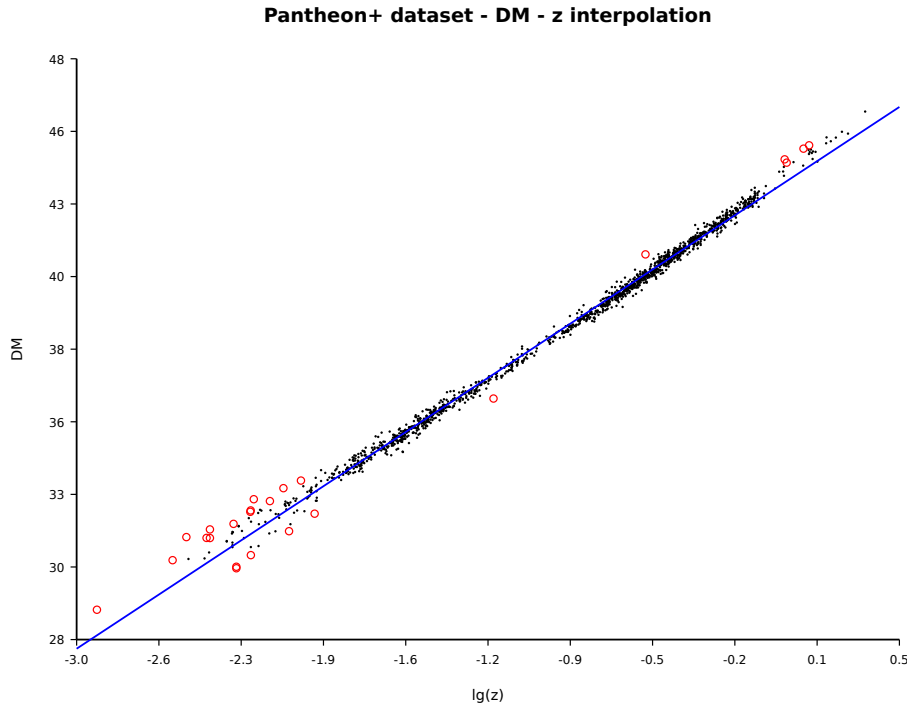


Figure 9: In the plot the correlation between distance modulus DM and $\log z$ directly from Pantheon+ database, the blue line represents the optimal linear interpolation, points in hollow red circles are outliers $> 3\sigma$.

The dispersion of data at low redshift probably depends on a combination of the factors considered in the cited studies. In particular, the Phillips relation (involved in the distance

modulus, see Eq. (21)), which links peak luminosity to the shape of the light curve, is particularly sensitive to peculiar effects (proper velocities, local conditions, and intrinsic variability). These at low redshift can become significant and introduce greater dispersion in the observed magnitudes and consequently calculated measurements.

This anomalous minority of deviations with greater dispersion remains within the range expected from the data quality, and are independent of the same hypothesized model, but the resulting distribution of residuals cannot be strictly Gaussian; see §9.3, due to long tails.

To address concerns regarding model sensitivity at low redshift, a comparative benchmark was set by fitting both the URD model and the standard Λ CDM model to the same Pantheon+ dataset. The analysis shows that in the local regime ($z < 0.01$, $N = 113$ Sn), both models exhibit an identical residual dispersion of approximately 20.5 % ($\sigma_{\text{URD}} = 20.61$ % vs $\sigma_{\Lambda\text{CDM}} = 20.41$ %, with a ratio of 1.01). Furthermore, this very good agreement on residual scatter persists in all redshift bins up to $z = 2.3$. These results prove that the high variance observed at low redshifts is an intrinsic property of the observational data rather than a sensitivity artifact of the URD model, as both the unconventional power-law and the standard cosmological frameworks respond identically to these data pathologies.

9.5 Comparison with Hubble–Lemaître law and the ‘Hubble Tension’

The distance D_{URD} estimated by Eq. (15) is not linearly dependent on the redshift, whereas the traditional Hubble–Lemaître law (Hubble 1929 [59], Lemaître 1927 [17]) is:

$$D_{H_0} = \frac{c}{H_0} z = \frac{c}{H_0} (\xi - 1) \quad [\text{Mpc}] , \quad (25)$$

where H_0 is the Hubble constant.

Therefore, necessarily the results cannot agree except for low redshift values, e.g. within the range:

$$z \leq 0.01 \quad \text{or} \quad \xi \leq 1.01 . \quad (26)$$

With the Hubble–Lemaître law in the ‘extended’ formulation to the second-degree term (see Ciufolini & Wheeler 1995 [60] p. 206) the luminosity distance is:

$$D_{H_0} = \frac{c}{H_0} \left[z + \frac{1}{2}(1 - q_0) z^2 + \dots \right] \quad [\text{Mpc}] , \quad (27)$$

where q_0 is the current estimated value of $q(t) \equiv -a(t)\ddot{a}(t)/\dot{a}^2(t)$, the deceleration parameter considered in the standard cosmological model¹⁵ (where $a(t)$ is the scale factor), usually estimated with the value $q_0 \approx -0.5$ (in a flat universe).

The Eq. (15), expressed in z , can be expanded in a Taylor series, in a right-hand neighborhood of $z = 0$:

$$D_{\text{URD}} = \frac{1}{\alpha_{U^*}} \left[z + \frac{1}{2}(1 + \delta_g) z^2 + \dots \right] \quad [\text{Mpc}] . \quad (28)$$

Comparing Eq. (27) and Eq. (28), we can see the similarity of the equations, but the correction parameter δ_g does not necessarily coincide with the cosmological deceleration parameter (even though they are similar), since the underlying model is different (see also §8.2). Furthermore, for low redshift values as in Eq. (26) where Eq. (27) can be applied, the second-order term becomes negligible, so the weight of the parameter vanishes. Considering the first degree terms (or the Eq. (16)), we can, however, derive a useful expression for the comparison of the value of α_{U^*} with the known values of the Hubble constant H_0 :

$$H_0 \approx c \alpha_{U^*} , \quad (29)$$

¹⁵In the standard cosmological model, $q_0 = \frac{1}{2} \Omega_{m,0} + \Omega_{r,0} - \Omega_{\Lambda,0}$ can be expressed as the weighed sum of the various components of the universe: $\Omega_{m,0}$ is the density parameter of matter (ordinary and dark); $\Omega_{r,0}$ is the density parameter of radiation; $\Omega_{\Lambda,0}$ is the density parameter of the cosmological constant (dark energy).

therefore, based on Eq. (24), we get the value:

$$H_0^{\text{URD}} = 72.45 \pm 0.01 \quad [\text{Km s}^{-1} \text{Mpc}^{-1}] . \quad (30)$$

We report some measurements from the recent literature, which highlight a discrepancy greater than 5σ in the measurements H_0 : we can summarize these evaluations into two groups of interpretations: considering late Universe (direct, distance ladder) and early Universe (indirect, model-dependent) methodologies.

- Late Universe (direct measurements). These values rely on geometric calibrations and standardized candles in the local cosmic volume
 - Cepheid-supernova Distance Ladder: Riess et al. 2024 [61]. Combining JWST Cepheid measurements in 19 hosts (24 Sne Ia) with HST data (37 hosts, 42 Sne Ia) yields $H_0 = 73.49 \pm 0.93 \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$. Including 35 TRGB-based calibrations (from HST and JWST) totals 55 Sne Ia and gives $H_0 = 73.18 \pm 0.88 \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$. 6σ above the cosmic microwave background expectation $\Lambda\text{CDM}+$.
 - The custom expanded photosphere method (EPM) applied to Sne II: Vogl et al. 2025 [62]. They find an independent value $H_0 = 74.9 \pm 1.9 \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$.
 - The Chicago-Carnegie program using JWST data: Freedman 2025 [63]. The paper presents the latest Hubble constant measurements. The best TRGB (Tip of the Red Giant Branch) estimate: $H_0 = 70.39 \pm 1.22(\text{stat}) \pm 1.33(\text{sys}) \pm 0.70 \sigma_{Sn} \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$; JWST-only TRGB result: $H_0 = 68.81 \pm 1.79(\text{stat}) \pm 1.32(\text{sys}) \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$; JWST-only JAGB (J-Region Asymptotic Giant Branch, an infrared region) result: $H_0 = 67.80 \pm 2.17(\text{stat}) \pm 1.64(\text{sys}) \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$.
 - Surface Brightness Fluctuations (SBF) method (an independent secondary distance indicator, validated across multiple photometric bands in wide-field surveys) [64]. The paper cites some works based on this method: Khetan et al. 2021 [65] $H_0 = 71.2 \pm 2.4(\text{stat}) \pm 3.4(\text{sys}) \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$; Blakeslee et al. 2021 [66] $H_0 = 73.3 \pm 0.7(\text{stat}) \pm 2.4(\text{sys}) \quad \text{km s}^{-1} \text{Mpc}^{-1}$; Garnavich et al. 2023 [67] report $H_0 = 74.6 \pm 0.9(\text{stat}) \pm 2.7(\text{sys}) \quad \text{km s}^{-1} \text{Mpc}^{-1}$.
- Early Universe (indirect measurements). These values are inferred by applying the standard ΛCDM model to the physics of the primordial plasma.
 - Cosmic Microwave Background (CMB): Planck Collaboration 2020 [68] $H_0 = 68.34 \pm 0.81 \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$
 - Atacama Cosmology Telescope (ACT), measurements of CMR in combination with measurements of BAO (Baryon Acoustic Oscillations) and Big Bang nucleosynthesis: Madhavacheril et al. [69]: $H_0 = 68.3 \pm 1.1 \quad [\text{km s}^{-1} \text{Mpc}^{-1}]$.

The set of all these values represents what is called the 'Hubble tension'. The Eq. (30) of this study, within the mathematical and theoretical limits considered, is rightly positioned among the values of direct measurements (and not indirect ones based on ΛCDM model).

10 Testing the URD equation and parameters with other datasets

In this Section, the proposed cosmological Eq. (15) and the phenomenological parameters Eq. (24) are tested against the BAO (Baryon Acoustic Oscillations) analysis, and other large datasets of 10,888 radio galaxies and 2,421 quasars.

10.1 Testing URD model with BAO ruler

The galaxy clustering data show a characteristic scale of agglomeration, commonly known as BAO, observed by telescopes across different redshifts. The observed constant dimensionless transverse ratios D_M/r_s estimates the average length for the standard ruler¹⁶ $r_s = 147.09 \pm 0.26$ Mpc (see Planck Collaboration 2020 [68], where D_M is the *comoving transverse distance*).¹⁷

Within the adopted phenomenological framework, there are currently no definitive and purely physical answers regarding the origin of this specific spatial structure, since it cannot be linked to acoustic waves in a primordial plasma of the *recombination era*. However, it is reasonable to conjecture that such a granularity of matter could be connected to the anisotropies of the CMB radiation.

To evaluate the URD model against *standard rulers* and perform a first-order consistency test, we consider the theoretical comoving transverse distance D_M , as being equivalent to the pure geometric distance $\tilde{D}_{\text{URD}}$ Eq. (14). In this static framework, D_M represents the spatial scale that is not affected by the time dilation components that characterize the measurements of the luminosity distance. Under a consistent geometric model, the ratio between this theoretical distance and the observed angular scale ratio of the BAO should yield a constant ruler value r_s across all redshifts.

A methodological caveat must be addressed: BAO data points reported in the literature (such as those compiled by DESI and SDSS) are typically presented as D_M/r_s ratios reconstructed under an assumed fiducial Λ CDM cosmology. This reconstruction, which involves converting observed angles $\Delta\theta$ and redshift intervals Δz into distances, introduces a mild model dependence (chiefly through the Alcock-Paczyński effect¹⁸; see Alcock & Paczyński 1979 [70]). However, since the URD distance profile closely tracks the standard comoving distance profile over the considered redshift range (as validated by the Pantheon+ calibration), comparing the reported ratios directly to $\tilde{D}_{\text{URD}}$ serves as a reliable first-order test of the geometric consistency of the framework.

The application of Eq. (14) to 15 high-precision BAO data points from the DESI Collaboration 2024 [5] and SDSS eBOSS datasets, using parameter values Eq. (24) derived from URD calibration with the Pantheon+ database, shows remarkable stability. As evidenced in Table 4, the calculated values URD_r_s remain constrained within a narrow range across redshifts spanning from the local universe to $z = 2.33$. The convergence of these measurements across 15 markers¹⁹, yields a mean $r_s = 138.37 \pm 6.60$ Mpc, characterized by a standard deviation of only 4.77%. A constant ratio across the redshift range ($0.15 \leq z \leq 2.33$) indicates that the URD model well reproduces the dimming history of the sources in the ‘not-so-far’ universe as measured by the cosmic standard ruler.

The fact that a single set of geometric parameters can simultaneously describe the brightness of Pantheon+ standard candles and the physical size of the acoustic horizon across nearly ten billion years of cosmic history provides empirical support for the consistency of the URD equation’s adequacy. The stability of the acoustic scale ratio implies that the URD model successfully accounts for the geometric description of the universe within the scope of the adopted phenomenological framework. This results in a unified description of the cosmos, where apparent

¹⁶In the standard model, r_s represents the maximum distance traveled by sound waves in primordial plasma at the time of separation between matter and light; in the URD model, it simply indicates a ‘fixed’ characteristic scale observed in the spatial distribution of galaxies.

¹⁷In the static, non-expanding Euclidean geometry of the URD model, the comoving transverse distance D_M is equivalent to the physical angular diameter distance D_A , such that $D_M = D_A = \tilde{D}_{\text{URD}}$. This differs from the standard expanding model where $D_M = (1+z)D_A$ due to the metric expansion of space.

¹⁸The Alcock-Paczynski test [70] quantifies the geometric distortion introduced when angular and radial separations are converted into distances using an incorrect cosmological model. Because the calibrated URD distance profile closely tracks the standard comoving distance profile over the range $0.15 \leq z \leq 2.33$, the resulting distortion parameter remains very close to unity. Consequently, the systematic bias introduced by the fiducial cosmology is negligible compared to the 4.77% statistical dispersion of the BAO dataset, see Table 4.

¹⁹In Son et al. 2025 [71] they consider fewer markers to analyze their universe expansion.

Table 4: URD BAO (Baryon Acoustic Oscillations) Analysis.

z	obs_ratio	URD_ D_M [Mpc]	URD_ r_s [Mpc]	Source
0.15	3.97	538.5	135.7	DESI 2024 BGS
0.15	4.47	538.5	120.5	SDSS MGS
0.38	10.23	1342.1	131.2	SDSS BOSS low-z
0.51	13.36	1796.6	134.5	SDSS BOSS high-z
0.51	13.62	1796.6	131.9	DESI 2024 LRG1
0.70	17.65	2348.6	133.1	SDSS eBOSS LRG
0.71	17.71	2376.4	134.2	DESI 2024 LRG2
0.85	20.21	2752.6	136.2	SDSS eBOSS ELG
0.93	21.71	2958.1	136.3	DESI 2024 LRG3
1.11	25.75	3398.2	132.0	DESI 2024 ELG1
1.32	27.79	3877.4	139.5	DESI 2024 ELG2
1.48	30.21	4221.1	139.7	SDSS eBOSS QSO
1.49	30.00	4242.1	141.4	DESI 2024 QSO
2.33	37.50	5811.0	155.0	SDSS Ly- α
2.33	39.71	5811.0	146.3	DESI 2024 Ly- α
Mean r_{drag} : 138.37 Mpc, Std Dev: 6.60 Mpc (4.77% <i>format</i>)				

z^* : the cosmological redshift of the BAO measurement. **obs_ratio***: observed dimensionless BAO scale ratio (D_M/r_s). **URD_** D_M : predicted comoving transverse distance (in Mpc). **URD_** r_s : the URD-derived BAO ruler length (in Mpc). **Source***: identifiers of the astronomical survey and tracer population. * means original dataset values.

discrepancies within the standard model are reinterpreted as natural consequences of light propagation within a non-expanding geometry.

See Appendix B Sections §12.5 and §12.6, for an interpretive conjecture regarding the standard rulers.

10.2 Testing URD model with large dataset in radio domain

To further test the Eq. (15) D_{URD} distance equation, in the radio domain, we consider a dataset²⁰ of 13,293 galaxies with available Tully–Fisher data (TF, see Tully & Fisher 1977 [49]); see `_file2.csv`, available for download, along with the other project files, on the dedicated page of our website: <https://andreabrussi.it/data-download>; see Table 5). The TF relation establishes a correlation between the luminosity of a spiral galaxy and its rotational velocity, typically measured via the 21 cm HI line width. This relation allows for the derivation of a distance modulus (DM_{TF}) independent of the redshift.

Initial analysis of the full dataset revealed significant anomalies (see Figure 10). At very low redshifts²¹ ($z < 0.005$), the derived distances are heavily affected by peculiar velocities (‘Hubble flow departures’), whereas at higher ranges ($z > 0.04$), the data shows increased dispersion and potential systematic biases in the TF calibration. These edge effects indicate that the global dataset contains data of questionable value for an accurate model comparison.

²⁰Extracted from Vizier database: <https://vizier.u-strasbg.fr/viz-bin/VizieR>; dataset J/ApJ/944/94

²¹A note on redshift: it is calculated for each galaxy from $\xi = 1 + z = V_{\text{CMR}}/c$, where c is the speed of light and V_{CMR} is the observational value for the galaxy obtained with TF mechanism; this is acceptable only because the sample has a low redshift, with $z < 0.04$, in the non-relativistic limit ($v \ll c$).

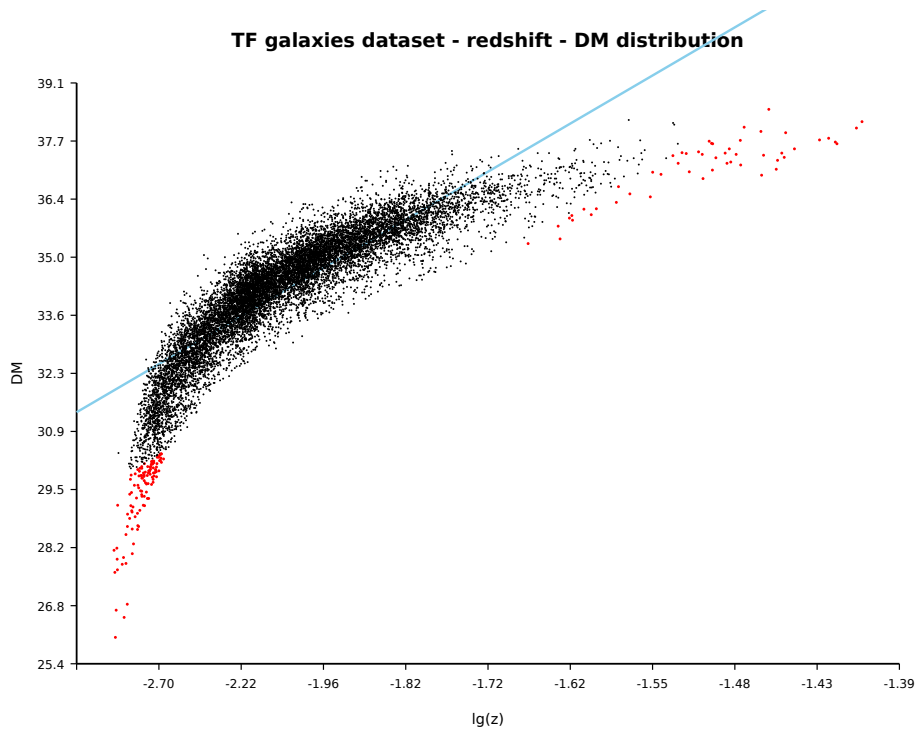
Table 5: Contents of `_file2.csv`: observational data and calculated fields for TF evaluated galaxies dataset J/ApJ/944/94 (header and first 5 records).

NUM	PGC	VCMR	DM_TF	DM_TF_ERR	XI	D_URD	D_DM_TF
1	2	4726	34.535	0.46	1.015764	65.8711	80.7235
2	4	4109	33.495	0.39	1.013706	57.1989	50.0035
3	12	6195	34.995	0.41	1.020664	86.6058	99.77
4	16	5312	34.655	0.45	1.017719	74.1277	85.31
6	55	4454	34.245	0.39	1.014857	62.0454	70.6318
...							

NUM*: original dataset numbering; **PGC***: Principal Galaxies Catalog ID; **VCMR***: Radial velocity; **DM_TF***: Distance modulus, Tully-Fisher relation; **DM_TF_ERR***: Uncertainty in DM_TF; **XI**: $(1+z)$ calculated from $XI = VCMR/c$; **D_URD**: URD distance based on XI, Eq. (15), and Eq. (24); **D_DM_TF**: distance based on DM_TF, and Eq. (23).

* means original dataset values.

Therefore a selection criterion of $0.005 < z < 0.04$ was applied to isolate the most reliable astronomical data. This filtering resulted in a sub-sample of 10,888 galaxies where the TF mechanism operates with high consistency (see Figure 11). Within this ‘clean’ interval, the distance modulus follows a stable linear trend, with a significantly reduced rmse of 0.489 in the central bin, corresponding to 0.55 mag. The performance of the URD Eq. (15) was evaluated against this selected dataset.

**Figure 10:** In the graph we have reported the whole J/ApJ/944/94 dataset (from VizieR database) of 13,293 galaxies with TF evaluation, revealing significant anomalies.

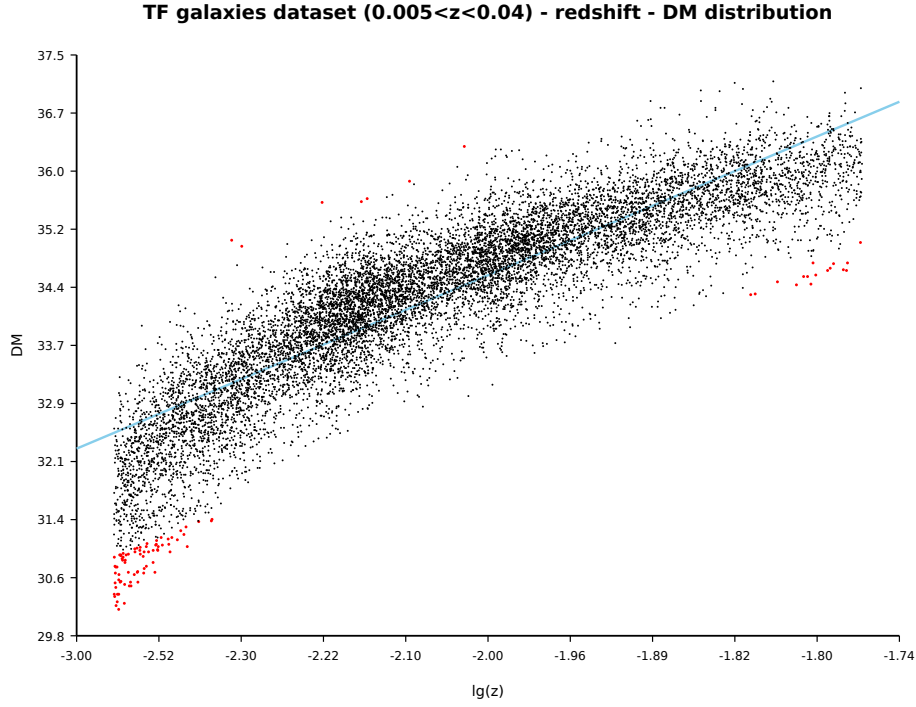


Figure 11: In the graph we have applied a selection criterion of $0.005 < z < 0.04$ to the whole J/ApJ/944/94 dataset (from VizieR database) of galaxies with TF evaluation, to isolate the most reliable astronomical data, resulting in a sub-sample of 10,888 galaxies; in red the outliers $> 3\sigma$.

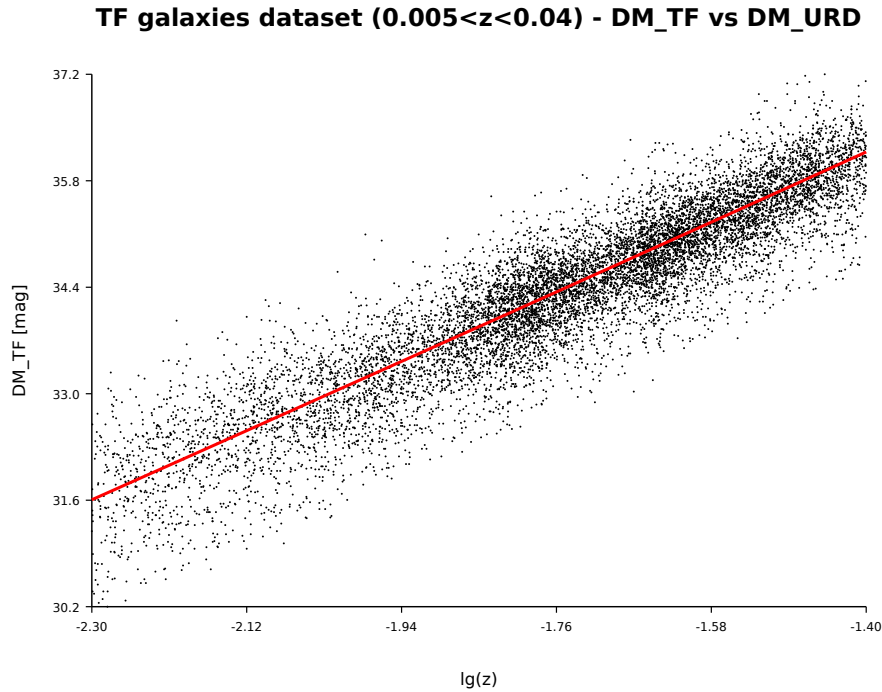


Figure 12: In the graph we have reported the trend D_{URD} of the Eq. (15) with respect to the distances D_{DM} derived from selected dataset ($0.005 < z < 0.04$) of galaxies with TF evaluation, within J/ApJ/944/94 dataset (from VizieR database), by means of the distance modulus Eq. (23).

A critical methodological refinement adopted in this analysis is the use of the DM scale for both statistical comparison and visualization. Testing the model directly on linear distances introduces a severe mathematical bias: because distance is related to DM via the exponential

Eq. (23), any small uncertainty in DM is exponentially amplified into a large dispersion on the linear scale of distances. This ‘enlargement’ effect is a purely coordinate-dependent artifact that can obscure the physical accuracy of the model. By comparing DM_TF directly with DM_URD, the accuracy of the model is preserved (see Figure 12).

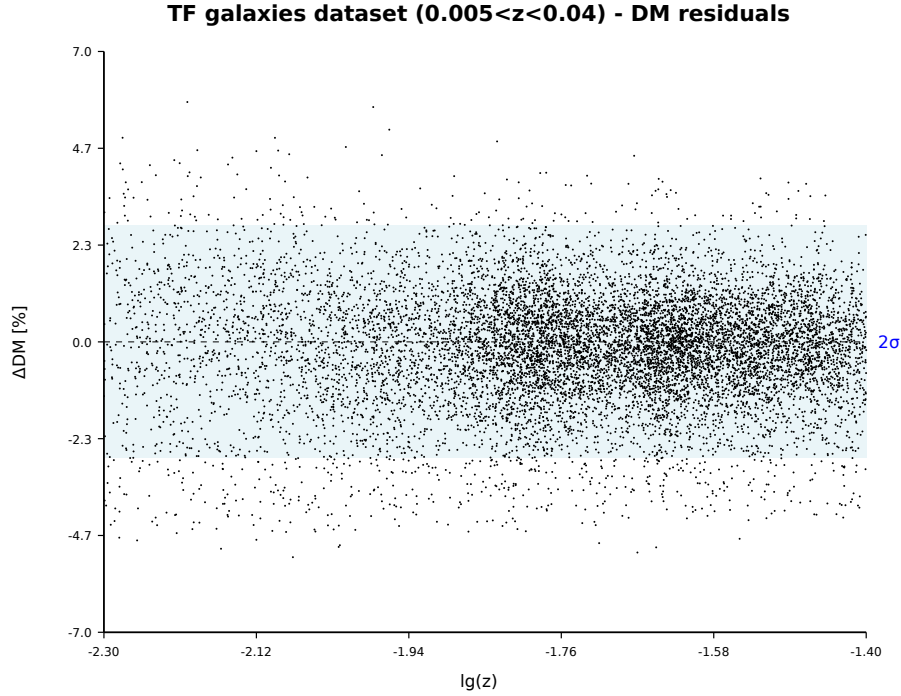


Figure 13: In the graph, the distribution of fractional distance residuals between the distances calculated with distance modulus derived from selected dataset ($0.005 < z < 0.04$) of TF evaluated galaxies, within J/ApJ/944/94 dataset (from VizieR database), and the ones calculated with URD model. The shaded blue band highlights the (2σ) observational uncertainty $\pm 43.36\%$.

Quantitative analysis of residuals, defined as $100(D_{\text{DM}} - D_{\text{URD}})/D_{\text{URD}}$, confirms the accuracy of the model; see Figure 13:

- Mean Residual: -0.18% or -0.063 mag (indicating that the model is perfectly centered on the average distance trend).
- Statistical Dispersion of the URD model with respect to data: 0.476 mag, corresponding to $\sim 1.4\%$ in the DM scale.

The fact that the model’s dispersion (0.476 mag) is slightly lower than the intrinsic dispersion of the data relative to a straight line (0.55 mag) suggests that the main cause of the final dispersion is not the URD model, but the intrinsic uncertainty of the TF data. The URD model effectively captures the trend in radio distances, aligning with the natural noise of the dataset without introducing significant systematic errors.

10.2.1 About TF calibration

A potential concern involves the inherent cosmological assumptions used in the initial calibration of the TF distance moduli. Since most astronomical datasets are pre-processed within the standard framework, a hidden dependence H_0 could be introduced into the comparison. However, for the redshift range considered in this specific test ($z < 0.04$), the distance-redshift relationship is dominated by the linear Hubble flow, where the differences between the URD model and the standard framework are mathematically minimal. Consequently, any assumed H_0 in the

TF dataset manifests itself primarily as a constant shift in the absolute distance scale (zero-point) rather than a bias in the geometric trend. This is precisely consistent with the observed -0.063 mag offset, which likely represents the residual difference between the specific H_0 assumed in the TF calibration and the α_{U^*} parameter derived independently in the URD framework. Since the same TF relation is based on the local physical correlation between rotational velocity and luminosity, its use here remains a valid probe of the model’s geometric consistency, with the small offset serving as a testament to the cross-calibration limit between independent distance indicators.

10.2.2 Deeper Statistical Insights

Quantitative deepening of the residuals for the sample of 10,888 galaxies included in the range $0.005 < z < 0.04$ provided the following results:

- Mean residual (ΔDM): -0.063 mag; this represents a systematic offset of only -0.18% in the DM scale, showing a remarkable alignment with the predictions of the URD model.
- Statistical dispersion $\sigma = 0.476$ mag (1.398% on the DM scale); the intrinsic variability of the TF relation is significantly low, allowing for a high-precision geometric consistency test.
- Reduced Chi-squared $\chi^2_{\text{red}} = 1.157$ (using observational errors only); a value so close to 1 indicates that the URD model accounts for the observed distances within the limits of measurement noise, without requiring additional parameters or systematic corrections.
- Skewness: -0.240 ; the distribution is nearly symmetric, with a slight tilt towards negative residuals, suggesting that the model captures the central tendency of the galaxy population across the local universe.
- Excess Kurtosis: $+0.575$. The distribution is slightly leptokurtic compared to a pure Gaussian, but remains stable and well-shaped, with few significant outliers.

Although the -0.063 absolute magnitude offset is small, its statistical significance 13.7σ indicates a formally detectable systematic bias that warrants discussion. This discrepancy likely originates from the technical differences between the independent datasets and their respective calibration procedures. A primary source could be a slight mismatch in the zero-point calibration between the TF radio galaxy sample and the optical Sn dataset used to calibrate the URD parameters. Additionally, inherent differences in photometric systems and the handling of Galactic extinction between radio and optical tracers can easily induce systematic shifts of this order (typically 0.1 mag). Furthermore, whereas the URD model captures the global cosmological trend with consistency, a minor systematic residual may emerge from applying a strictly geometric equation to complex rotational-velocity-based indicators. These factors, combined with the massive sample size that amplifies even minimal calibration offsets into high statistical significance, suggest that the bias reflects the limit of cross-calibration between different distance indicators rather than a structural flaw in the URD framework.

10.3 Testing URD model with large dataset of quasars

To test the URD distance Eq. (15) for high-energy radiation, we analyzed the extensive catalog²² compiled by Lusso et al. 2020 [72]; this dataset (see `_file3.csv`, available for download, along with the other project files, on the dedicated page of our website: <https://andreabrussi.it/data-download>; see Table 6) includes 2,421 quasars with spectroscopic redshifts and distance

²²Extracted from VizieR database: <https://vizier.u-strasbg.fr/viz-bin/VizieR>; dataset J/A+A/642/A150.

modulus values derived from the non-linear relationship between their rest-frame ultraviolet (2500 Å) and X-ray (2 keV) monochromatic luminosities of the sources²³.

Table 6: Contents of `_file3.csv`: observational data and calculated fields for quasars dataset J/A+A/642/A150 (header and first 5 records).

NUM	QNAME	z	DM_Q	DM_Q_ERR	D_URD	DM_URD
1	030341.04-002321.9	3.2350	46.4238	0.3084	29714.20	47.3648
2	030449.85-000813.4	3.2960	45.6192	0.2042	30495.67	47.4212
3	090508.88+305757.3	3.0340	44.9859	0.1741	27184.71	47.1716
4	094202.04+042244.5	3.2840	46.5461	0.3301	30341.43	47.4102
5	094734.19+142116.9	3.0340	46.4477	0.2055	27184.71	47.1716
...						

NUM: record number; **QNAME***: quasar name; **z***: Redshift; **DM_Q***: Distance modulus from UV–X-ray nonlinear relation; **DM_Q_ERR***: Uncertainty in **DM_Q**; **D_URD**: URD distance based on $\xi = 1 + z$, Eq. (15) and Eq. (24); **DM_URD**: distance module based on **D_URD** and Eq. (20). * means original dataset values.

This empirical correlation is expressed as $\lg(L_X) = \gamma \lg(L_{UV}) + \beta$, where L_X and L_{UV} are the luminosities (energy emitted) in the bands X e UV, γ and β are two parameters calibrated with Sn analysis for the same sources at medium redshift.²⁴ This equation allows for the determination of the luminosity distance directly from the observed fluxes (F_X and F_{UV}). By relating this equation to the absolute luminosity L , the luminosity distance D_L , and the observed flux F , they obtain: $\lg(D_L) = [\gamma \lg(F_{UV}) - \lg(F_X) + \beta'] / 2(1 - \gamma)$, where $\beta' = \beta + (\gamma - 1) \lg(4\pi)$ combines all constant parameters. This luminosity distance D_L was then converted by the authors into the corresponding distance module **DM_Q** reported in the dataset. Following the standard approach in quasar cosmography, the statistical analysis in this work is performed directly in terms of the distance modulus, which is the primary quantity provided in the VizieR dataset. Indeed, the transition from the logarithmic distance modulus (**DM_Q**) back to the linear distance (see Eq. (23)) introduces a severe mathematical bias: any small uncertainty in the modulus is exponentially amplified in linear distance, leading to an ‘explosive’ dispersion that obscures the physical trend. Consequently, the rigorous way to validate the geometric consistency of the URD model at extreme high- z is directly on the DM scale, where the uncertainty range is preserved. This choice ensures that these observational uncertainties (**DM_Q_ERR**) propagate symmetrically, preserving the statistical reliability of the cosmological fits. For this reason, the calculated distance **D_URD** has been converted to its corresponding distance modulus **DM_URD**.

The application of the URD model to this dataset is of particular interest as it provides a rigorous test for the equation across a vast range of redshift values, reaching as high as $z = 7.5$ and in an electromagnetic band far from the optical band used to calibrate the URD distance relation (parameters Eq. (24)). By comparing the predictions of the URD equation with the distances derived from the internal physics of quasar accretion disks and coronae, we can evaluate the consistency and robustness of the model across the furthest reaches of the observable

²³This evaluation method was introduced in the paper Lusso and Risaliti 2016 [73], then subsequently improved.

²⁴The use of the Lusso et al. 2020 [72] quasar dataset involves a degree of calibration circularity, as the parameters of the L_X – L_{UV} relation (γ and β) were originally anchored to the SNIa distance scale assuming a standard cosmological framework. This legacy calibration means that the absolute distance moduli of the quasars carry an implicit model dependence. However, the primary value of the quasar sample in this comparison lies in its ability to test the geometric progression of the URD model at extreme redshifts (z up to 7.54), far beyond the supernova anchor range. Whereas the global $-0.54 \text{ mag}/z$ trend observed in the residuals likely reflects this initial calibration mismatch between the URD power-law and the standard framework, the stability of the fit at high redshift indicates that the geometric expansion effectively tracks the physical trend of the data. Acknowledging these caveats is essential for a balanced interpretation, confirming that whereas the absolute normalization may be inherited from the standard calibration, the URD model shows a consistent geometric effectiveness for the high-redshift universe.

domain, providing a comprehensive consistency that extends far beyond the limits of traditional supernova-based samples.

10.3.1 Distribution Analysis and Redshift-induced Bias

The initial analysis of the entire sample (2,421 sources) focused on the raw distribution of the distance modulus (DM_Q) versus $\lg(z)$, as shown in Figure 14. The sample shows a significant statistical dispersion (rmse) of -0.133 mag (-0.262%), which characterizes the high intrinsic variability of these active galactic nuclei.

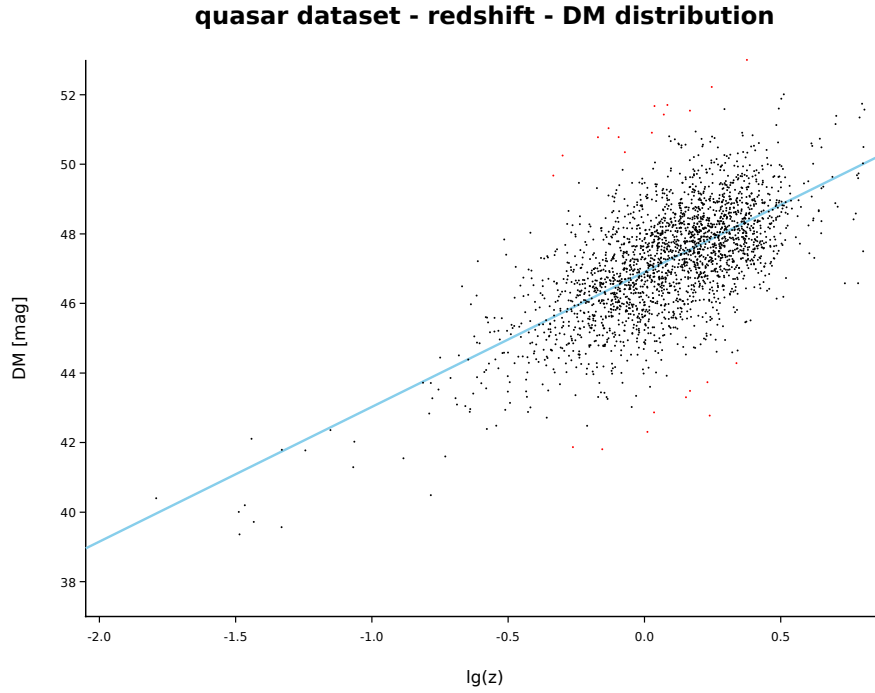


Figure 14: In the graph we have reported the whole dataset of 2,421 quasars from VizieR database; the red dots mark outliers $> 3\sigma$.

10.3.2 Results and Model Comparison

The comparison between the measured DM_Q and the theoretical predictions of URD (DM_{URD}) is illustrated in Figure 15. The URD model accurately tracks the central geometric mean of the dataset throughout the redshift range.

The stability of the model was further validated through the analysis of percentage residuals:

$$\Delta DM [\%] = 100 \frac{DM_Q - DM_{URD}}{DM_{URD}} .$$

The final residuals plot (see Figure 16) confirms a mean percentage residual of -0.1822% (corresponding to an offset of -0.0971 mag) and a consistent statistical dispersion ($\sigma = 3.615\%$ or 1.6067 mag).

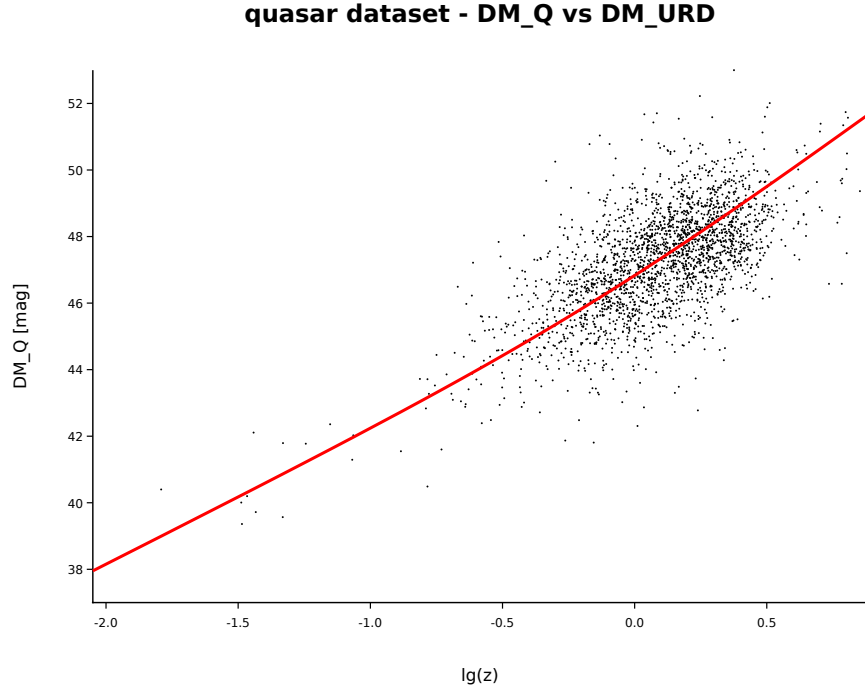


Figure 15: In the graph we have reported the trend DM_URD calculated with the parameters Eq. (24) and the Eq. (15) with respect to the distances DM_Q derived from quasar J/A+A/642/A150 dataset (from VizieR database).

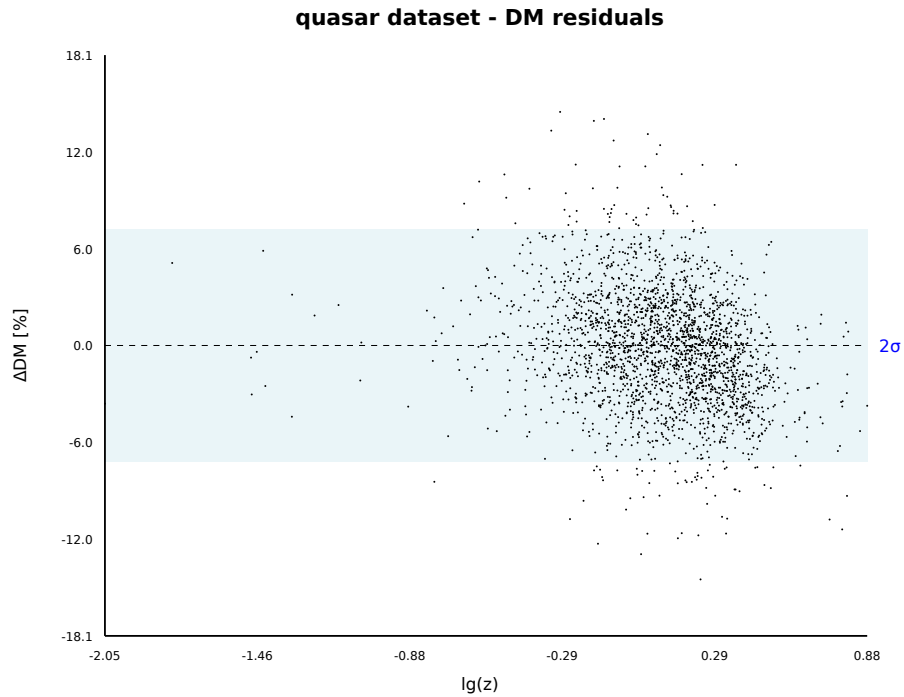


Figure 16: In the graph, the distribution of fractional distance moduli residuals between quasar of J/A+A/642/A150 dataset (from VizieR database), and the ones calculated with URD model. The shaded blue band highlights the (2σ) observational uncertainty $\pm 7.23\%$.

10.3.3 Deeper Statistical Insights

Quantitative deepening of the residuals provided the following results:

- Reduced Chi-squared $\chi_{\text{red}}^2 = 47.01$, using observational errors only; this indicates that the residuals are overwhelmingly dominated by the intrinsic physical variance of the sources rather than by observational measurement noise.
- Skewness: $+0.1344$; the distribution is remarkably symmetric, suggesting that the model tracks the geometric ‘spine’ of the quasar sample without strong systematic drift.
- Excess kurtosis: $+0.0189$; the leptokurtic nature of the sample explains the presence of significant outliers whereas maintaining central stability.
- Statistical importance: 2.97σ .

An analysis of 2,421 quasars shows that the high-redshift subset (504 sources with $z > 2$) does not exhibit any specific anomalous trend beyond the global behavior of the dataset. Although the entire quasar sample shows a systematic negative trend of $\sim -0.54 \text{ mag} / z$ relative to the supernova-calibrated URD model, this trend actually stabilizes for the sources at $z > 2$, where the slope is reduced to $\sim -0.35 \text{ mag} / z$. The statistical dispersion remains consistent at 1.42 mag across the high-redshift range. These results indicate that the URD model successfully tracks the geometric backbone of the high-energy universe without encountering explosive drift at extreme redshifts, and the observed deviations are inherited from the intrinsic global physical properties of the quasar distance calibration.

In conclusion, although the quasar dataset presents much higher intrinsic noise compared to supernovae or TF galaxies, the URD distance equation maintains a good degree of geometric consistency (within $\sim 0.3\%$) even at extreme UV and X-ray energies up to $z = 7.5$. This reliability could be put to the test by future observations from instruments such as the JWST or even more advanced ones, at even higher redshifts. This suggests a significant predictive capability for the URD distance equation, one to be verified!

11 Conclusions

The URD framework is formulated on a set of working assumptions (§1–2, Appendix B §13) that define a consistent interpretive structure for the observational data considered, in particular redshift and Cosmic Microwave Radiation (CMR) (§3–4). These assumptions are grounded in established physical principles, such as energy conservation, which are preserved within the proposed description.

Within this framework, the progressive energy attenuation of propagating photons is modeled as a continuous, conformal geometric dissipation within an extended gravity ($f(R)$) spacetime (§6). This mechanism, driven by the non-minimal coupling of the electromagnetic field with the background scalar curvature (which we consider physically sourced by the CMR energy density U^*), is treated as an effective geometric process that naturally preserves photon directionality and achromaticity.

The resulting redshift-distance relationship D_{URD} , expressed by Eq. (15), has been calibrated using the Pantheon+ dataset (§9.1), and its statistical properties and predictive capacity have been analyzed. The increased dispersion at low redshift in the Pantheon+ sample has also been examined and interpreted within the framework (§9.4). Known challenges associated with photon energy-loss models (often referred to as *tired light*, §7) have been re-evaluated and addressed within the URD approach (time dilation §8.3.1), Tolman test §8.3.2–8.3.4).

The D_{URD} relation has been further tested against independent datasets, including baryon acoustic oscillations (BAO, §10.1), galaxies measured by the Tully–Fisher relation (§10.2) and

high-redshift quasars (§10.3). In all cases, good agreement is found between the URD predictions and the observational data over a wide redshift range, extending up to $z = 7.5$.

Taken together, these results indicate that a non-expansion-based phenomenological description of the redshift–distance relation can reproduce current observational constraints across multiple datasets. In a broader context where open observational tensions continue to motivate phenomenological investigations, the present work may contribute to the exploration of alternative interpretations alongside the standard framework. For example, possible geometric refinements, such as geodesic corrections explored within the URD model, are likely relevant for data interpretation.

More work is ongoing to extend both the observational consistency and the theoretical foundations of the proposed framework.

Note: The author declares that no funds, grants, or other support were received during the preparation of this manuscript.

12 Appendix A — On the Cosmic Microwave Radiation conjecture

The framework in which the CMR conjecture on microwave radiation is conceived is that of the premises of this study, which we briefly recall here: the universe we know developed within an infinitesimal part of a spatially infinite universe, in a low-entropy context, and in global thermodynamic equilibrium, with no initial singularity, no inflation, and no subsequent expansion. Space is on average not curved except locally due to the presence of matter-energy and is on average homogeneous and isotropic. The diffuse energy undergoes an attenuation phenomenon observed as redshift, which in fact limits the knowable part, which remains infinitesimal with respect to the whole. Since there are no temporal constraints, the diffuse radiation that is observed is not caused only by the evolution of matter-energy in the local context but is the diffuse radiation coming from the entire universe. One does not fall into Olbers' paradox (Olbers 1823 [74]) because the extension is infinite but the density of matter-energy is not, and the redshift of diffuse radiation from the evolution of matter-energy (stars, galaxies, supernovae, etc.) limits its observability.²⁵

Some explanations supporting a non-standard qualitative conjecture, consistent or compatible with these premises, are detailed in the following Section.

12.1 Space as an energetic superfluid (inspired by Fiscaletti & Sorli 2017 [75])

Instead of a geometric vacuum in expansion, the universe is modeled as an infinite physical medium with the properties of a superfluid in a critical state; in this framework, space-time is not an abstract manifold but a dynamic 'Quantum Vacuum' with a non-zero Planck energy density:

- **Physical Support:** this medium acts as the fundamental substrate for the propagation of all radiation. The speed of light c is interpreted as the constant phase velocity of waves within this non-dispersive superfluid.
- **Stationary Universe:** unlike the Λ CDM model, this approach suggests that redshift and cosmic features arise from the intrinsic properties of the medium rather than the metric expansion of a void (this could be an alternative interpretation to the universe as a black body generating CMR, see the following §12.2).

12.2 Origin of the microwave radiation (CMR)

The CMR is not the echo of an initial explosion, but the permanent energy deposit of the conformal dissipation process, both 'local', not only in the meaning of the universe knowable from our point of view, but even from adjacent regions.

²⁵In the URD model, the observability of the infinite universe is constrained by DURD Eq. (15), which defines a wavelength-dependent physical horizon. Assuming a maximum detectable wavelength of $\lambda_{\max} \approx 10^4$ m (beyond which photons merge with quantum vacuum fluctuations), we find that measurements of the 'visible' universe depend on the initial wavelength of the emitted photons:

- *High-energy limit (γ -rays):* for a source emitting at $\approx 10^{-14}$ m, the redshifted signal reaches the $\approx 10^4$ m threshold at a distance of approximately $1.7 \cdot 10^{33}$ light-years; this represents the ultimate physical boundary for the most energetic phenomena in an infinite static universe.
- *Radio limit (21 cm line):* for the neutral hydrogen signal ($\lambda \approx 0.21$ m), the redshifted limit is reached at $6.6 \cdot 10^{11}$ light-years.

Beyond these distances, signals are not merely redshifted but are physically extinguished as information-bearing entities, dissolving into the stochastic background. However, it is conceivable that over such immense distances, a photon beam from a source—even one as large as a galaxy—could undergo progressive energy decay due to cumulative geometric dissipation, alongside gravitational perturbations that could alter the direction and morphology of the original signal, rendering it too weak to be detected even at shorter distances. This mechanism effectively resolves Olbers' Paradox (Olbers 1823 [74]) and replaces the temporal Big Bang horizon with a geometric-energetic horizon, where the apparent 'beginning' of the observable universe is simply the point of total information decay from the observer's perspective.

The CMR at a temperature of 2.7 K represents the thermodynamic equilibrium point of the medium, where the diffuse energy of propagation losses accumulates and levels out. Metaphorically, we could call it an ‘energetic dew point’.

Perfect black body: the black body shape of the CMR is ensured by the continuous mixing and thermalization of the dissipated energies within the (infinite) superfluid medium.

12.3 Justification of anisotropies and ‘granularity’

The CMR anisotropies can be interpreted through self-organized criticality as a structured ‘quantum fluid’; this perspective, supported by theories of fractal structures and quantum electrodynamic coherence domains, describes the universe as a self-regulating, scale-invariant system with emergent physical properties:

- Self-Organized Criticality (SOC): following the principles established by Bak 1996 [76], the cosmic medium exists in a ‘critical state’; the continuous interaction between energy sources and the background maintains a dynamic equilibrium, generating fluctuations across all scales.
- Fractal Coherence Domains: the 1-degree anisotropies are interpreted as the largest coherence domains (Preparata 1995 [77]) or ‘vortices’ within the superfluid; these represent the upper limit of a fractal subdivision of energy (Pietronero et al. 1987 [78]).
- Emergent Fine Structure: below these large-scale domains, an increasingly fine granularity exists, linked to individual quantum fluctuations within the vacuum; whereas current observational limits focus on the largest ‘bubbles’ of coherence, the SOC model implies a continuous spectrum of fluctuations reaching down to the Planck scale.

12.4 Explanation of acoustic peaks in a static universe

In this framework, the peaks observed in the CMR power spectrum are not ‘fossils of a primordial explosion’ (from the *recombination* era). Instead, they represent the current vibrational response and harmonic resonance of the space-energy medium (quantum vacuum).

The amplitude and angular position of these peaks are determined by the intrinsic physical properties of the cosmic superfluid through three synergistic mechanisms:

12.4.1 Jeans length of the quantum vacuum (gravity-pressure equilibrium)

In a superfluid medium with a non-zero energy density, a critical scale exists where internal pressure balances gravitational attraction (inspired by Fiscaletti 2016 [79]).

- Mechanism: in a static, infinite universe, the vacuum self-organizes into standing waves at this characteristic length; the ‘first peak’ of the CMR corresponds to the fundamental frequency of this resonance, dictated by the local energy density of the space-fluid; this replaces the *acoustic scale* of the standard model with a stationary wave horizon intrinsic to the vacuum structure.

12.4.2 Structural Resonance and Harmonic Modulation

The distribution of matter (galaxies and filaments) is not random but follows a periodic, fractal network (Pietronero et al. 1987 [78]).

- Mechanism: the CMR does not travel through an absolute void but through a medium modulated by the Large-Scale Structure (LSS) of the universe; the subsequent peaks in the spectrum are interpreted as higher harmonics generated by the interaction (scattering and thermalization) of background radiation with the cosmic web; in this view, the CMR acts

as a ‘resonance chamber’ that reflects the current geometry of visible matter, rather than a past event.

- Key Concept: the interaction between photons and the vacuum’s ‘inertial cycles’ creates a harmonic signature that matches the spatial periodicity of the galaxy distribution (consistent with the plasma universe models of Alfvén 1986 [80]).

12.4.3 Standing waves in the non-dispersive medium

Since the cosmic medium is non-dispersive and the velocity of propagation c is constant, energetic pressure waves generated by localized astrophysical sources (such as stellar collapses or galactic dynamics) do not simply dissipate. Instead, in an infinite and static universe, they undergo continuous superposition.

- Mechanism: over immense timescales, these overlapping wavefronts create a stable interference pattern or a system of standing waves that permeate the entire quantum vacuum.
- Interpretation of data: what modern cosmology interprets as *acoustic peaks* from a primordial era is, in this model, the resonance spectrum of this ‘ocean of energy’; the peaks represent the preferred frequencies at which the cosmic superfluid vibrates, determined by its fundamental density and the spatial distribution of matter (consistent with the Fluid Quantum Vacuum model in Fiscialetti 2016 [79])
- Steady-state observation: because the timescales of these cosmic vibrations far exceed human observation, the pattern appears static to our instruments, providing a deceptive ‘snapshot’ that is mistaken for a relic of a singular past event.

12.5 A speculative hypothesis for future research on BAO–CMR tension and the 5/3 Kolmogorov scaling

We have made some speculative assumptions (for future investigation) based on certain studies of the ‘BAO–CMB tension’, finding a similar value that could, in fact, simply be a coincidence. Modern cosmological surveys, particularly the latest DESI BAO Collaboration 2024 [5] data combined with ACT observations (see Madhavacheril et al. 2024 [69]), have identified a significant statistical tension that challenges standard inflationary models such as the Starobinsky inflation plateau (see Starobinsky 1980 [29] and his subsequent papers). This ‘BAO–CMB tension’ is resolved by adopting polynomial α -attractor models (Ferreira et al. 2026 [81]), which point toward a theoretical scaling target $\beta = 5/3$. This exact numerical result might suggest a connection for the URD framework, as the 5/3 exponent is the fundamental signature of a *Kolmogorov cascade* in fluid systems (Kolmogorov 1941 [82]), with a trend toward increasingly smaller structures, as suggested —albeit in vague terms— in §12.3 and §12.4 Sections above. Recent indications from precision cosmology (Ferreira et al. 2026 [81]) toward this specific scale suggest that the harmonic features detected in the cosmic microwave background and in baryon distributions may not be remnants of a primordial past, but may instead manifest as a turbulent energy cascade within the self-organized cosmic medium interpreted by the URD framework.

12.6 Conclusions on CMR conjecture

This conjecture describes the infinitesimal part of an eternal, static, self-regulated universe that evolved into the universe knowable to us today. It is a premise and an integration of the URD framework, defining the framework on which it is based. A proof of this model’s consistency would lie in the observation of fractal granularity in the CMR, which would persist even below current resolution limits. Unlike the Big Bang model, which predicts a blur limit (Silk damping, see Silk 1967 [83]) due to diffusion (beyond a certain point one should observe only random noise),

the URD framework implies a continuous energy cascade with dissipation toward increasingly smaller structures, consistent with the Kolmogorov scaling discussed in the previous Section §12.5. The conjecture also provides a physical justification for the integrated Sachs–Wolfe effect (Sachs & Wolfe 1967 [84]); whereas the standard model relies on dark energy, the observed temperature variations in correspondence with large voids and clusters are interpreted here as direct indications of the metric’s response to local mass-energy densities under extended gravity.

13 Appendix B — A framework for a spatially infinite, non-time-limited, globally static cosmology

The framework within which the URD framework is developed is a globally flat, spatially infinite, and non-time-limited²⁶ spacetime. It challenges the standard application of the FLRW metric, specifically the assumption of a dynamical scale factor $a(t)$, by reinterpreting GR as a strictly local phenomenon within a non-coherent matter distribution. In this framework, the observed flatness of the universe is a fundamental geometric property rather than a dynamical result of inflation, and the cosmological redshift is modeled as a cumulative energy dissipation process rather than metric expansion.

13.1 Geometric foundations: global flatness and spatial infinity in static cosmology

Standard cosmological models require inflationary mechanisms to explain the observed spatial flatness of the universe. In contrast, our conjecture posits that spacetime is globally flat and spatially infinite as a baseline geometric condition. In an unbounded, open Euclidean manifold, flatness is not a state requiring fine-tuned dynamical balancing but the inherent structural property of the vacuum. Although local mass-energy distributions induce curvature according to the Einstein field equations, these perturbations do not coalesce into a global curvature. Specifically, we distinguish between local geodesic propagation and the asymptotic global behavior of the manifold. On local and intermediate scales, discrete mass-energy distributions induce localized curvature according to the Einstein field equations ($G_{\mu\nu} \neq 0$). These local perturbations affect light propagation, which requires the introduction of our geodesic correction parameter (δ_g) to accurately compute distances.

However, on macroscopic (cosmological) scales, these localized perturbations average out. Consequently, the universe is modeled as a manifold where the statistically averaged Einstein tensor $\langle G_{\mu\nu} \rangle$ approaches zero. This scale-dependent transition explains why local geodesic corrections are physically necessary, while the global geometry remains consistent with the large-scale Euclidean spatial flatness ($\Omega_k \approx 0$) observed by the Planck 2020 [68] and DESI 2024 [5] Collaborations.

13.2 The rejection of the continuum approximation in static cosmology

A central premise of this model is the rejection of the global ‘fluid approximation’ used in standard cosmology. By treating the universe as a homogeneous and isotropic continuous medium, the FLRW solutions necessarily imply a global expansion or contraction of the metric.

This framework instead treats the universe as a non-coherent continuum, in which physical processes are effectively confined within localized causal domains. Whereas matter and energy are continuously distributed, the finite propagation of interactions and the progressive degradation of

²⁶Regarding the *entity time in Nature*, phenomena occur when circumstances determine them, a simple tautology that describes reality, *Time causes nothing*, but in Nature, the principle of causality is in force, which can be understood as a Boolean sequence (see Boole 1847 [85]), which could be called ‘logical time’ that determines and regulates the evolution of systems.

signals prevent the establishment of a globally synchronized dynamical behavior. As a result, the universe cannot be treated as a single, coherent dynamical system. The absence of a global center of mass further implies that there is no privileged direction or point toward which a global collapse could occur.

It is important to clarify that the rejection of the fluid approximation in this context refers specifically to the standard cosmological treatment of matter as a structureless, continuous medium governing global metric dynamics. This does not contradict the hypothesis of a quantum superfluid-like vacuum discussed in Appendix A §12.1. In the present framework, the term 'fluid' is not used in the thermodynamic sense of a perfect, homogeneous medium but rather to describe a structured space-energy substrate characterized by coherence, self-organized criticality, and scale-invariant fluctuations. In this sense, the standard perfect-fluid approximation is replaced by a non-trivial physical medium, whose properties emerge from underlying microphysics rather than from a phenomenological equation of state.

Within this interpretation, the large-scale structure of the universe emerges from local interactions, whereas the underlying spacetime remains globally static and non-evolving.

13.3 Local dynamics vs global stasis in static cosmology

Although GR remains valid for describing the evolution of local systems, this model distinguishes between local gravitational aggregation and global metric evolution.

- Local aggregation: gravity drives the formation and eventual collapse of stars and galaxies within their respective causal domains.
- Global stasis: the underlying spacetime metric does not undergo a universal stretching or contraction.

The 'beginning' of the universe (Big Bang) is reinterpreted as an observational limit imposed by the finite speed of light and the dissipation of information (see the note 25), rather than a temporal singularity.

13.4 Temporal nature: a non-time-limited static universe

The universe is described as not time-limited, rejecting the notion of a singular temporal origin. The standard estimate of the 'age of the universe' (~ 13.8 Gyr) is viewed as the detection horizon of current instrumentation, the point at which signals from distant sources become indistinguishable from background noise (see the note 25).

By removing the temporal constraint of a 13.8 Gyr origin, the existence at high redshift of highly evolved, 'dead' elliptical galaxies is no longer a chronological paradox. These structures have had an unlimited duration to evolve, exhaust their low-entropy fuel, and reach their end-states within a globally static spacetime. In this view, the universe is *always* 'in progress', and the fact that certain infinite-scale processes have not 'completed' is a natural consequence of their divergent timescales²⁷.

13.5 Redshift as a cumulative energy horizon in static cosmology

In the URD framework, cosmological redshift is decoupled from the recession velocity of galaxies. Instead, it is modeled as a progressive, physical loss of energy experienced by photons as they traverse vast cosmic distances.

²⁷To address the paradox of why the universe has not reached a state of maximized metallicity in a non time-limited frame, we must again distinguish between the global cosmos and our local context. What we measure as the 'age of the universe' (and its initial low-entropy state of hydrogen and helium) represents the temporal origin of only our infinitesimal local region. In an infinite space, localized domains undergo their own independent evolutionary cycles. Thus, the existence of highly evolved galaxies at high redshift is consistent with the localized timeframe of our specific cosmic 'bubble', without requiring global temporal boundaries.

This cumulative energy decay establishes a physical detection horizon: as the wavelength of propagating photons increases, it eventually reaches a threshold where the signal-to-noise ratio prevents further observation. This dissipation provides a self-consistent, geometric-energetic explanation for the dark night sky (resolving (Olbers' paradox, Olbers 1823 [74]) and accounts for the observed redshift-distance relationship without requiring a dynamical expansion of the spatial metric. The quantitative boundaries of this horizon and its wavelength dependence are detailed in footnote 25.

13.6 Causal disconnection and statistical uniformity in static universe

The universe as a whole can be described as a continuum that is locally connected but globally non-coherent (see Section §13.2). Although local physical processes follow the same fundamental laws, they do not coordinate their evolution.

The finite speed of propagation of 'any signal' (c for massless signals), combined with the progressive degradation of energy, prevents local perturbations from synchronizing into a global dynamical mode. In physical terms, this corresponds to an effective absence of global normal modes. Therefore, the universe behaves as an ensemble of effectively independent thermodynamic domains. In this context, the observed large-scale uniformity is not attributed to early-universe thermal equilibrium; instead, it is regarded merely as an empirical property of our observable domain, requiring no global dynamical origin.

13.7 Conclusions about static cosmology

As introduced in §1, this cosmological framework offers an alternative viewpoint to the Big Bang model by interpreting the universe as a spatially infinite, non-time-limited, and globally static ensemble: it preserves the local predictive power of GR without the support of dark energy, inflation, and a temporal singularity. The cosmos is thus viewed not as a singular evolving entity, but as a vast, non-coherent system of locally evolving domains within a stable, flat, and infinite spacetime. The phenomenological study, conducted against observational data, was developed within the framework of this working hypothesis.

References

- [1] Jonathan P. Gardner, John C. Mather, Mark Clampin, Rene Doyon, Perry Greenfield, Ronald Gilliland, Heidi B. Hammel, Jim Humphreys, Peter Katz, Marc J. Kuchner, Matthew Lallo, Charles R. Lawrence, Stefano Meschiari, Matt Mountain, Andrew Nell, Franco Pacini, Marcia Rieke, James Sherman, George Sonneborn, Massimo Stiavelli, H. S. Stockman, Rogier A. Windhorst, and Gillian S. Wright. The james webb space telescope. *Space Science Reviews*, 123(4):485–606, 2006.
- [2] S. S. McGaugh, J. M. Schombert, F. Lelli, and J. Franck. Accelerated structure formation: The early emergence of massive galaxies and clusters of galaxies. *Astrophys. J.*, 976:13, 2024.
- [3] Katherine Wright. JWST sees more galaxies than expected. *Physics Magazine*, APS, 2024. Accessed: 2025.
- [4] E. di Valentino et al. The CosmoVerse white paper: Addressing observational tensions in cosmology with systematics and fundamental physics. *Phys. Dark Univ.*, 49:101965, 2025.
- [5] DESI Collaboration. DESI 2024 III: Baryon acoustic oscillations from galaxies and quasars. *J. Cosmol. Astropart. Phys.*, 04:012, 2025.
- [6] N. Lovyagin et al. Cosmological model tests with JWST. *Galaxies*, 10:108, 2022.

- [7] E. J. Lerner et al. UV surface brightness of galaxies from the local universe to $z \sim 5$. *Int. J. Mod. Phys. D*, 23:1450058, 2014.
- [8] Eric J. Lerner. Observations contradict galaxy size and surface brightness predictions that are based on the expanding universe hypothesis. *Monthly Notices of the Royal Astronomical Society*, 477(3):3185–3196, 2018.
- [9] Christopher J. Conselice, Edmund J. Copeland, and Sergio Sevilano Muñoz. The revised evolutionary volume tolmán test: Cosmological constraints from galaxy evolution. *arXiv preprint*, 2026.
- [10] Rodríguez-Beltrán, P., Cerviño, M., Vazdekis, A., and Beasley, M. A. Modelling of surface brightness fluctuation measurements - methodology, uncertainty, and recommendations. *Astronomy & Astrophysics*, 686:A62, 2024.
- [11] Elcio Abdalla et al. Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies. *Journal of High Energy Astrophysics*, 2022.
- [12] Lior Shamir. An empirical consistent redshift bias: A possible direct observation of zwicky’s theory. *Particles*, 7(3):703–716, 2024.
- [13] F. Zwicky. On the redshift of spectral lines through interstellar space. *Proc. Natl. Acad. Sci.*, 15:773–779, 1929.
- [14] P. Tremblin and G. Chabrier. Re-evaluating the cosmological redshift: Insights into inhomogeneities and irreversible processes. *Astronomy & Astrophysics*, 689:A207, 2024.
- [15] Michael Aaron Cody. Cosmological redshift without expansion. *Preprints*, 2025.
- [16] A. Friedmann. Über die krümmung des raumes. *Z. Phys.*, 10:377–386, 1922.
- [17] G. Lemaître. Un univers homogène de masse constante et de rayon croissant rendant compte de la vitesse radiale des nébuleuses extra-galactiques. *Ann. Soc. Sci. Bruxelles*, A47:49–59, 1927.
- [18] H. P. Robertson. Kinematics and world-structure. *Astrophys. J.*, 82:284–301, 1935.
- [19] A. G. Walker. On Milne’s theory of world-structure. *Proc. Lond. Math. Soc.*, s2-42:90–127, 1936.
- [20] Robin Booth. The origins of redshift in a static universe. *Preprints*, 2024.
- [21] Albert Einstein. Die grundlage der allgemeinen relativitätstheorie. *Annalen der Physik*, 354(7):769–822, 1916.
- [22] C. Doppler. Über das farbige licht der doppelsterne und einiger anderer gestirne des himmels. *Abh. königl. böhm. Ges. Wiss.*, 1842.
- [23] A. A. Penzias and R. W. Wilson. A measurement of excess antenna temperature at 4080 mc/s. *Astrophys. J.*, 142:419–421, 1965.
- [24] S. Muller, A. Beelen, J. H. Black, S. J. Curran, C. Horellou, S. Aalto, F. Combes, M. Guélin, and C. Henkel. A precise and accurate determination of the cosmic microwave background temperature at $z = 0.89$. *Astron. Astrophys.*, 551:A109, 2013.
- [25] M. Planck. Über eine verbesserung della Wienschen spektralgleichung. *Verhandl. Dtsch. Phys. Ges.*, 2:237, 1900.

- [26] D. J. Fixsen et al. The cosmic microwave background spectrum from the full COBE FIRAS data set. *Astrophys. J.*, 473:576, 1996.
- [27] G. Kirchhoff. Ueber das verhältniss zwischen dem emissionsvermögen und dem absorptionsvermögen der körper für wärme und licht. *Ann. Phys.*, 185:275–301, 1860.
- [28] S. N. Bose. Plancks gesetz und lichtquantenhypothese. *Z. Phys.*, 26:178–181, 1924. See also Einstein, A., Sitzungsber. Preuss. Akad. Wiss., 1924.
- [29] A. A. Starobinsky. A new type of isotropic cosmological models without singularity. *Phys. Lett. B*, 91:99–102, 1980.
- [30] Salvatore Capozziello. Curvature quintessence. *International Journal of Modern Physics D*, 11(04):483–491, 2002.
- [31] Salvatore Capozziello, Vincenzo F. Cardone, S. Carloni, and Antonio Troisi. Curvature quintessence match with observational data. *International Journal of Modern Physics D*, 12(10):1969–1995, 2003.
- [32] Salvatore Capozziello and Mariafelicia De Laurentis. Extended theories of gravity. *Physics Reports*, 509(4–5):167–321, 2011.
- [33] A. G. Riess et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron. J.*, 116:1009–1038, 1998.
- [34] S. Perlmutter et al. Measurements of Ω and Λ from 42 high-redshift supernovae. *Astrophys. J.*, 517:565–586, 1999.
- [35] Edward L. Wright. Errors in tired light cosmology. <https://www.astro.ucla.edu/~wright/tiredlit.htm>, 2008. Accessed: Oct 15, 2025.
- [36] R. C. Tolman. On the estimation of distances in a curved universe with a non-static line element. *Proc. Natl. Acad. Sci.*, 16:511–520, 1930.
- [37] G. Goldhaber et al. Timescale stretch parameterization of type ia supernova b-band light curves. *Astrophys. J.*, 558:359–368, 2001.
- [38] R. A. Hutchin. The size of a photon. *Opt. Photon. J.*, 11:121–131, 2021.
- [39] R. M. T. White, T. M. Davis, G. F. Lewis, D. Brout, L. Galbany, K. Glazebrook, S. R. Hinton, J. Lee, C. Lidman, A. Möller, M. Sako, D. Scolnic, M. Smith, M. Sullivan, B. O. Sánchez, P. Shah, M. Vincenzi, P. Wiseman, T. M. C. Abbott, et al. The dark energy survey supernova program: Slow supernovae show cosmological time dilation out to $z \sim 1$. *arXiv preprint*, 2024.
- [40] I. H. Stairs. Testing general relativity with pulsar timing. *Living Rev. Relativ.*, 6:5, 2003.
- [41] M. Kramer et al. Tests of general relativity from timing the double pulsar. *Science*, 314:97–102, 2006.
- [42] W. Mattig. Über den zusammenhang zwischen rotverschiebung und scheinbarer helligkeit. *Astron. Nachr.*, 284:109–111, 1958.
- [43] L. M. Lubin and A. Sandage. The tolmán surface brightness test for the reality of the expansion. IV. a measurement of the tolmán signal and the luminosity evolution of early-type galaxies. *Astron. J.*, 122:1084–1103, 2001.

- [44] J. L. Sérsic. Influence of the atmospheric and instrumental dispersion on the brightness distribution in a galaxy. In *Bol. Asoc. Argentina Astron.*, volume 6, page 41, 1963.
- [45] G. de Vaucouleurs. Recherches sur les nébuleuses extragalactiques. *Ann. Astrophys.*, 11:247, 1948. Bibcode: 1948AnAp...11..247D.
- [46] V. Petrosian. Surface brightness and evolution of galaxies. *Astrophys. J. Lett.*, 209:L1–L5, 1976.
- [47] D. Brout et al. The pantheon+ analysis: Cosmological constraints. *Astrophys. J.*, 938:110, 2022.
- [48] A. Carr et al. The pantheon+ analysis: Improving the redshifts and peculiar velocities of type ia supernovae used in cosmological analyses. *Publ. Astron. Soc. Aust.*, 39:e046, 2022.
- [49] R. B. Tully and J. R. Fisher. A new method of determining distances to galaxies. *Astron. Astrophys.*, 54:661–673, 1977. Bibcode: 1977A&A...54..661T.
- [50] M. M. Phillips. The absolute magnitudes of type ia supernovae. *Astrophys. J. Lett.*, 413:L105–L108, 1993.
- [51] H. Theil. A rank-invariant method of linear and polynomial regression analysis. In *Henri Theil's Contributions to Economics and Econometrics*, pages 345–381. Springer Netherlands, 1992. Originally published in *Indagationes Math.*, 12, 85–91, 1950.
- [52] B. Efron. Bootstrap methods: Another look at the jackknife. *Ann. Statist.*, 7:1–26, 1979.
- [53] A. G. Riess et al. A 2.4% determination of the local value of the Hubble constant. *Astrophys. J.*, 826:56, 2016.
- [54] G. Schwarz. Estimating the dimension of a model. *Ann. Statist.*, 6:461–464, 1978.
- [55] R. E. Keeley, A. Shafieloo, and B. L’Huillier. An analysis of variance of the Pantheon+ dataset: Systematics in the covariance matrix? *Universe*, 10:439, 2024.
- [56] E. R. Peterson et al. The pantheon+ analysis: Evaluating peculiar velocity corrections in cosmological analyses with nearby type ia supernovae. *Astrophys. J.*, 938:112, 2022.
- [57] R. Wojtak and J. Hjorth. Intrinsic tension in the supernova sector of the local Hubble constant measurement and its implications. *Mon. Not. R. Astron. Soc.*, 515:2790, 2022.
- [58] Leandros Perivolaropoulos and Foteini Skara. On the homogeneity of snia absolute magnitude in the pantheon+ sample. *Monthly Notices of the Royal Astronomical Society*, 520(4):5110–5125, 2023.
- [59] E. Hubble. A relation between distance and radial velocity among extra-galactic nebulae. *Proc. Natl. Acad. Sci.*, 15:168–173, 1929.
- [60] I. Ciufolini and J. A. Wheeler. *Gravitation and Inertia*. Princeton University Press, 1995.
- [61] Adam G. Riess et al. Jwst observations reject unrecognized crowding of cepheid photometry as an explanation for the hubble tension at 8σ confidence. *The Astrophysical Journal Letters*, 962, 2024.
- [62] C. Vogl et al. No rungs attached: A distance-ladder-free determination of the hubble constant through type ii supernova spectral modelling. *Astronomy & Astrophysics*, 702:A41, 2025.

- [63] Wendy L. Freedman et al. Status report on the chicago-carnegie hubble program (cchp): Measurement of the hubble constant using the hubble and james webb space telescopes. *arXiv preprint*, 2025.
- [64] Michele Cantiello and John P. Blakeslee. Surface brightness fluctuations. *arXiv preprint*, 2023.
- [65] Nandita Khetan, Luca Izzo, Marica Branchesi, Radosław Wojtak, et al. A new measurement of the hubble constant using type ia supernovae calibrated with surface brightness fluctuations. *Astronomy & Astrophysics*, 647:A72, 2021.
- [66] John P. Blakeslee, Joseph B. Jensen, Chung-Pei Ma, Peter A. Milne, and Jenny E. Greene. The hubble constant from infrared surface brightness fluctuation distances*. *The Astrophysical Journal*, 911(1):65, 2021.
- [67] Peter Garnavich et al. Connecting infrared surface brightness fluctuation distances to type ia supernova hosts: Testing the top rung of the distance ladder. *The Astrophysical Journal*, 953(1):35, 2023.
- [68] Planck Collaboration. Planck 2018 results. VI. cosmological parameters. *Astron. Astrophys.*, 641:A6, 2020.
- [69] Mathew S. Madhavacheril et al. The atacama cosmology telescope: Dr6 gravitational lensing map and cosmological parameters. *The Astrophysical Journal*, 962(2):113, 2024.
- [70] Charles Alcock and Bohdan Paczyński. An evolution free test for non-zero cosmological constant. *Nature*, 281(5730):358–359, 1979.
- [71] Junhyuk Son, Young-Wook Lee, Chul Chung, Seunghyun Park, and Hyejeon Cho. Strong progenitor age bias in supernova cosmology – II. alignment with DESI BAO and signs of a non-accelerating universe. *Mon. Not. R. Astron. Soc.*, 2025. arXiv:2510.13121.
- [72] E. Lusso et al. Quasars as standard candles. III. validation of a new sample for cosmological studies. *Astron. Astrophys.*, 642:A150, 2020.
- [73] E. Lusso and G. Risaliti. The tight relation between x-ray and ultraviolet luminosity of quasars. *The Astrophysical Journal*, 819(2):154, 2016.
- [74] H. W. M. Olbers. Über die durchsichtigkeit des weltraums. *Astron. Jahrb.*, 1823.
- [75] D. Fisaletti and A. Sorli. Quantum vacuum energy density and unifying perspectives between gravity and quantum behaviour of matter. *Ann. Fond. Louis Broglie*, 42:251, 2017.
- [76] P. Bak. How nature works: The science of self-organized criticality. *Copernicus Press*, 1996.
- [77] Giuliano Preparata. *QED Coherence in Matter*. World Scientific, Singapore, 1995.
- [78] L. Pietronero, M. Montuori, and F. Sylos Labini. On the fractal structure of the visible universe. *Critical Dialogues in Cosmology*, 1997. arXiv:astro-ph/9611197.
- [79] Davide Fisaletti. About a three-dimensional quantum vacuum as the ultimate origin of gravity, electromagnetic field, dark energy ... and quantum behavior. *Ukr. J. Phys.*, 61:413–431, 2016.
- [80] H. Alfven. Plasma universe. *Phys. Scr.*, T18:20–28, 1987.
- [81] Elisa G. M. Ferreira, Evan McDonough, Lennart Balkenhol, Renata Kallosh, Lloyd Knox, and Andrei Linde. BAO–CMB tension and implications for inflation. *arXiv preprint*, 2025. arXiv:2507.12459.

- [82] A. N. Kolmogorov. The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers. *Dokl. Akad. Nauk SSSR*, 30:301–305, 1941.
- [83] J. Silk. Fluctuations in the primordial fireball. *Nature*, 215:1155–1156, 1967.
- [84] R. K. Sachs and A. M. Wolfe. Perturbations of a cosmological model and angular variations of the microwave background. *Astrophys. J.*, 147:73, 1967.
- [85] G. Boole. *The Mathematical Analysis of Logic*. Macmillan, 1847.