

$$S, T \subset \mathbb{R} \mid \begin{cases} S \cup T = \mathbb{R} \\ \text{non vuoto} \\ \text{se } x \in S \text{ e } t \in T \rightarrow x < t \end{cases} \quad \begin{matrix} [1] \\ [2] \end{matrix}$$

$$\text{Din } \exists, \text{ unico } \beta \in \mathbb{R} \mid \begin{cases} x < \beta \rightarrow x \in S [3] \\ x > \beta \rightarrow x \in T [4] \end{cases}$$

$$\textcircled{\exists} \quad T \neq \emptyset \Rightarrow \forall \tilde{t} \in T \rightarrow \forall x \in S \Rightarrow x < \tilde{t} \quad \text{DA [2]},$$

ANCHE $S \neq \emptyset$ ED ESSENDO SUPERIORE
LIMITATO $\rightarrow \exists$ ESTREMO SUP $= \beta \in \mathbb{R}$

CONSIDER (LA [3]) $x < \beta$, ESSENDO β ESTREMO SUP

$$\rightarrow \exists \alpha \mid x < \alpha \leq \beta, \quad \text{DOBBIAMO DIR } x \in S.$$

$$\text{SUPPONENDO (MURDO)} \quad \left. \begin{matrix} x \in T \rightarrow \text{DA [2]} \Rightarrow \\ \end{matrix} \right\} \begin{matrix} \alpha \in S \\ t = x \in T \end{matrix}$$

MA SUPERBARE $S < x$, CONTRO

$$\rightarrow x \notin T \quad \text{MA } S \cup T = \mathbb{R} \Rightarrow x \in S$$

SIMILMENTE SI DIRA [4] CIOE' L'ESISTENZA

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SUPPONIAMO $\exists \alpha \in \mathbb{R}$

$$\text{E COSE } [3] \in [4] \quad \begin{cases} x < \alpha \rightarrow x \in S \\ x > \alpha \rightarrow x \in T \end{cases}$$

CASO $\alpha > \beta \quad \exists x \mid \beta < x < \alpha \mid x < \alpha \rightarrow x \in S$

$$\text{MA } x > \beta \rightarrow x \in T$$

QUINDI $x \in S \cap T$ NO! SAREBBE $x \in x$ DAU[2]

CASO $\alpha < \beta$ IDEN

QUINDI UNICO CASO POSSIBILE $\alpha = \beta$ CHE' UNICITÀ

Q.E.D.