

HYPOTHESIS

$$[1] S \neq \emptyset \neq T$$

$$[2] S \cup T = \mathbb{R}$$

$$[3] \forall \begin{cases} x \in S \\ t \in T \end{cases} \rightarrow x < t$$

THESIS $\exists, \text{unique } \beta \in \mathbb{R} \mid \forall x \in \mathbb{R} \begin{cases} \beta - x \in S \\ \beta + x \in T \end{cases}$

DEMONSTRATION

THEOREM 1.1.3 $\Rightarrow \exists, \text{unique } \sup S$
 " 1.1.8 $\Rightarrow \exists, \text{unique } \inf T$

REDUCTIO AD ABSURDUM

CASE 1) $\sup S > \inf T \rightarrow \exists x > t$ NO BY [3]

CASE 2) $\sup S < \inf T \rightarrow \exists \varepsilon \in \mathbb{R} \mid \sup S + \varepsilon < \inf T$

BUT THIS WAY COULD $\exists$
 REAL NUMBER $\sup S + \varepsilon \notin S \cup T$
 CONTRA [2] $\Rightarrow$ NO!

$\Rightarrow \sup S = \inf T = \beta$

HENCE THE THESIS, FOR 1.1.3, 1.1.8